ESTAR INTERVENTION

Tier 2 Mathematics Intervention

Module: *Multiplication & Division Relationships (MDR)*

Teacher Lesson Booklet

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

The Meadows Center

FOR PREVENTING EDUCATIONAL RISK

THE UNIVERSITY OF TEXAS AT AUSTIN

COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 1

Skip Counting Review

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will skip count by 2s, 3s, and 5s using a hundreds chart.
• The student will identify equal groups and write a repeated addition expression to show skip counting.
• The student will use language related to skip counting to explain reasoning and solutions.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">equal: the same amount
equal groups: objects separated into groups that have the same amount or quantity</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">skip counting, even, odd, row, equation, expression</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 1-7)
• Counters (about 20)
• Colored pencil
• Whiteboard with marker</td><td>• Student Booklet (pp. 1-4)
• Colored pencils or highlighters (1 per student)</td></tr>
</table>

Preview

Say: **Today we will skip count using the hundreds chart and identify groups and patterns.**

Engage Prior/Informal Knowledge — Time: 3 min

Discuss the idea of skip counting as a means for finding an answer quickly and efficiently.

Say: **Skip counting is counting using a pattern or jumping over some numbers. What is skip counting?** *(counting using a pattern)*

What is an example of skip counting? *(allow a variety of answers such as 5, 10, 15)*

When might you skip count? *(when you have many objects to count, when multiplying)*

What can skip counting help us do? *(count faster, count more groups of objects, find an answer quickly)*

Let's skip count by 10s. *(skip count by 10s with the students)*

What do we call the numbers like 10 and 20, even or odd? *(even numbers)*

What do you know about odd and even numbers? *(answers will vary)*

What makes a number even? *(even numbers can be divided equally into groups of 2)*

Modeled Practice — Time: 8 min

1. Review counting and making groups of 2s as an efficient way to find a total amount instead of counting by 1s.

   Spread 20 counters on the table. Ask a student to count each counter to find out the total amount.

Say: **I need to find how many counters are on the table. Count each piece to find the total.**

When the student reaches 12 counters, stop the student.

Say: **Counting each piece 1 at a time takes a while. Is there a faster way to count these counters instead of 1 by 1?** *(answers may vary but focus on counting by 2s)*

Help me put the counters into groups of 2 while we skip count by 2s to find the total quickly.

Each group has the same or *equal* amount. *Equal* means the same amount. We just put the counters into *equal groups*. The term *equal groups* means each group has the same amount. How many counters are in each *equal group*? *(2)* **What did we just skip count by?** *(2s)*

How many total counters? *(20)* **How many groups of 2?** *(10)*

2. Use the hundreds chart to identify patterns in skip counting by 2s.

Have students turn to the ***Modeled Practice Sheet*** and distribute a colored pencil or highlighter to each student.

As they skip count by 2s, students will lightly shade they numbers they say aloud on the hundreds chart to recognize the sequence of numbers when skip counting. Work along with the students, shading the hundreds chart as the lesson progresses.

Say: **Now we will use the hundreds chart to show the groups or the jumps. When we skip count by 2s we jump over a number or skip it.**

We are going to shade as we count by 2s on the hundreds chart until we reach 20. *(allow a few moments for students to shade)*

Do you see any patterns when we skip count by 2s? *(the ones place is always 2, 4, 6, 8, 0; all even numbers in the columns)*

With skip counting we are adding or making groups with the same amount each time. How much are we adding to each number to find the next number in the sequence? *(2)* **If we were making groups, how many counters would be in each group?** *(2)*

Skip counting is like adding: we started with 1 group of 2 and then added. How many were in the second group? *(2)*

Below your hundreds chart write 2 + 2. Did we add another group of 2? *(yes)* **How many total *equal groups* of 2 did we add together or skip count by?** *(10)*

The addition equation to show skip counting by 2s 10 times, or 10 groups of 2, is: 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20.

Teacher Note
For students who do not see the repetition of the digits in the ones place, bring their attention to the sequence. Have students look down the ones column starting at 6. Ask students what digit is in the ones place. Repeat with other columns.

Say: **At the beginning of the lesson we counted the 20 counters by 2s. Were all 20 counters put into groups of 2?** *(yes)* **Do you think we can put these counters into *equal groups* of 3, using all of them?** *(allow a variety of answers, such as 3 is odd and 40 is even so no)* **Try it. Make *equal groups* of 3.**

Is 3 an even number or odd? *(odd)* **We know that 3 is an odd number because all the counters could not be placed into *equal groups* of 2.**

To count by 2s we put 2 counters into each group, making *equal groups*. To count by 3s, how many counters should be in each *equal group*? *(3)*

Have students place the counters into 6 groups with 2 leftover.

Say: ***Equal* means the same. How many counters are in each group?** *(3)* **Can we count these last 2 counters as a group?** *(no)* **Why?** *(each group has to have 3)*

Think about when we counted the *equal groups* of 2 and shaded in the hundreds chart. All the counters were in a group and the total amount is shaded on the hundreds chart. We will shade the 3s next during Practice, but first think about 20. Will 20 be shaded? *(no)* **How do you know?** *(not in an equal group of 3)*

Practice — Time: 8 min

Activity 1: First read the problem and then skip count by 3s on the hundreds chart.

Have students turn to the *Practice Sheet* on page 2. Using the colored pencil or a highlighter, have students use the hundreds chart on the practice sheet.

Say: **First read the problem. Ready, read: "Makayla was given a sequence of numbers: 16, 20, 24, 28, 32. She knows that each number cannot be separated into groups of 3 or 5 equally. How could she prove this using the hundreds chart?"**

We need to figure out what Makayla was skip counting by. Was she counting by 2s? *(no)* **How do you know?** *(does not match the 2s hundreds chart we shaded in, skipping over more than 2 numbers)*

In the top row of the hundreds chart, start at 3 then add 3 more each time to shade the 3s sequence.

Shade in the numbers as you count by 3 in the top 4 rows. Stop when you reach 39.

What are the first 3 numbers in the skip counting sequence of 3? *(3, 6, 9)*

Was Makayla skip counting by 3s? *(no)* **How do you know?** *(not all those numbers were shaded)* **What number was she skip counting by?** *(4s)* **How do you know?** *(allow a variety of answers)* **Can you show it using the hundreds chart?** *(yes)*

Check that students have shaded in only the numbers as they skip count by 3 through 39. Students should be lightly shading so that the numbers on the page can still be seen.

Say: **What do you notice about the patterns of 3?** *(they go up or increase by 3, they jump over 2 numbers, they are both even and odd, accept other reasonable answers)*

Do you notice any patterns when looking at even and odd numbers when skip counting by 3s? *(yes, odd, even, odd, even, odd)*

How many numbers did we shade in the sequence of 3s? *(13)* **If we were making *equal groups*, how many objects would be in each group?** *(3)* **How many *equal groups*?** *(13)*

Watch For	**Students may confuse the terms even and odd. For example, students may state 3 is an even number. If students are unable to identify a number as even or odd provide a short review. Refer students to the 2s hundred chart. All the multiples of 2 are even numbers. All the numbers that are not a multiple of 2 are odd.**

Say: **Find 15 on the hundreds chart. Counting by 3s, how many groups of 3 is 15?** *(5 groups)*

How would I write the addition expression to show that we added together 5 groups of 3? *(3 + 3 + 3 + 3 + 3)*

We stopped shading at 39. What number is next when skip counting by 3s? *(42)* **How do you know?** *(counted 3 more)*

Have students shade all the numbers as they skip count by 3 on the hundreds chart before continuing.

Ask questions and give instructions such as:

- What interesting patterns do you notice on the hundreds chart once skip-counting sequences by 3 have all been shaded? *(the groups of 3 follow diagonal lines)*
- Do you use the lines to help you identify the next number in the sequence? *(allow student to verbalize their own method for finding groups)*
- Pick any shaded number from the hundreds chart and tell me how many groups of 3 it is. *(possible answer: 36 is 12 groups of 3)*
- What would be the addition expression? *(3 + 3…)*

Have students turn to the Practice Sheet on page 3. Have students complete the questions with a partner or small group and discuss the answers together.

Activity 2: Have students turn to the ***Practice Sheet*** on page 2. Have students practice skip counting by 5s on the hundreds chart. Use the same mathematical language and questions as students shade the sequence of 5s on the same hundreds chart.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 2

Equal-Groups Models for Multiplication

Lesson Objectives	• The student will identify number expressions that are and are not equal. • The student will use an equal-groups model to represent repeated addition. • The student will use language related to equal groups to describe how objects are separated into groups of the same amount.	
Vocabulary	**equal-groups model:** a diagram that shows a number or set of objects separated into equal parts **repeated addition:** a situation in which all groups have the same number of objects and are added together (e.g., 3 + 3 + 3)	
Reviewed Vocabulary	equal, equal groups, equation, expression	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 8-19) • Whiteboard with marker • Connecting cubes/cubes (15)	• Student Booklet (pp. 5-10) • Whiteboard with marker (1 per student) • Connecting cubes/cubes (15 per student) • Number cube (1 per student pair)

Preview

Say: **Today we will make equal groups and find its relationship to addition and multiplication.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students practice identifying equal groups and use skip counting to find how many in all. If available, use a balance to show equal groups by building the groups on either side.

Place 14 connecting cubes or cubes on the table.

Say: **Here is a pile of cubes. What is the faster way to count the total number of cubes, by 1s or 2s?** *(by 2s)*

The faster way to find the total number of cubes is to skip count by 2s.

Make groups of 2 and count the cubes with students.

Say: **Ready, count.** *(2, 4, … 14)*

How many cubes are there in all? *(14)*

How many cubes are in each group? *(2)*

Is there the same number of cubes in each group? *(yes)* **These are called equal groups.**

How many equal groups are there? *(7 groups)*

We have 7 equal groups of 2.

Can we make equal groups of 5 with 14 cubes? *(no)*

Have the students put the cubes intro groups of 5.

Say: **How many equal groups of 5 can you make?** *(2)*

How many cubes are left? *(4)*

Can we think of these 4 as an equal group? *(no)*

Why? *(the groups are not equal, the groups are not the same, the groups each need 5 or the same number of cubes)*

What if I added 1 more cube? Could we make another group of 5? *(yes)*

Add 1 cube.

Say: **How many equal groups are there now?** *(3)*

How many cubes are in each group? *(5)*

Can we skip count to find the total? *(yes)*

What number do we skip count by? *(5)*

How do you know to count by 5s? *(there are 5 in each group)*

Ready, count: 5, 10, 15. How many cubes are there in all? *(15)*

If time allows, have students make equal groups of cubes from 12 – 20 and skip count to find the total.

Modeled Practice

Time: 8 min

1. Represent and discuss the meaning of equal and the equal sign.

Tell students to put 12 connecting cubes in 1 group. Use the whiteboard.

Say: **We want to place these 12 cubes in equal groups.**

Equal means having the same amount or is the same as. What does equal mean? *(having the same amount or is the same as)* **Think of the equal sign as a balance. Whatever amount is on 1 side must be the same as the amount on the other side to be equal.**

Teacher Note
If available, use a balance to emphasize equal.

Write the "=" sign on the whiteboard and point to it.

Say: **What does this symbol mean?** *(equal)*

Does equal mean the answer? *(no)*

Write 1 + 5 = 3 + 3 on the whiteboard.

Look at the first expression. What is 1 + 5? *(6)* **Connect 6 cubes and place under 1 + 5. Write "6" below the expression.**

Look at the second expression. What is 3 + 3? *(6)* **Connect 6 cubes and place under 3 + 3. Write "6" below the expression. Are 6 and 6 equal?** *(yes)* **Write an equal sign between the 6s.**

Place the 2 groups of cubes next to each other. Is 1 + 5 equal to or the same amount as 3 + 3? *(yes)* **Why?** *(because 1 + 5 is 6 and 3 + 3 is 6; there is the same amount on both sides of the equal sign - 6)*

What does equal mean? *(having the same amount or is the same as)* **Both sides may not look the same but both sides equal the same amount; they are equal to each other.**

Erase your whiteboards. Write the expression 1 + 4 = 2 + 4. What is 1 + 4? *(5)* **Write "5" under the expression. What is 2 + 4?** *(6)* **Write "6" under the expression. Is this expression true, is 1 + 4 equal to 2 + 4?** *(no)* **How do you know?** *(because it is not equal, 5 is less than 6)*

2. Build equal-groups models and write repeated addition equations for each model.

Allow students time to make groups.

Say: **Put your 12 cubes into equal groups of 3.**

How many in each group? *(3)*

How many equal groups? *(4)*

Does each group have the same amount? *(yes)*

If we group the 12 cubes differently into 4 groups of 3 are the groups equal? *(yes, because they have the same amount in each group)*

When the 12 cubes are in equal groups we have an *equal-groups model,* which is a set of objects separated into equal groups.

Group the 12 cubes into 2 groups, 1 group of 8 and 1 group with 4.

Does this show an *equal-groups model?* *(no)*

Why? *(each group does not have the same amount)*

Is there another way to make equal groups with the 12 cubes? *(accept a variety of answers such as 6 groups of 2, 4 groups of 3, 2 groups of 6)*

Write "2 groups of 6" on the whiteboard. Build 2 groups of 6 while the students build along with you. Check that each student has built the model correctly. Provide immediate corrective feedback when needed.

Say: **Does your model have 2 groups with 6 cubes in each group?** *(yes)*

We can write an addition expression to represent the equal groups. What is the addition expression? *(6 + 6)*

When you have an addition expression with the same number being added it is called *repeated addition*.

Write 6 + 6 on your whiteboard. What is 6 + 6? *(12)* **Write = 12.**

Read the *repeated addition* equation for this *equal-groups model.* *(6 + 6 = 12)*

A *repeated addition* equation is an addition equation with all the groups having the same amount or number of objects.

Arrange the cubes in 3 groups of 4.

What is the *repeated addition* equation? *(4 + 4 + 4 = 12)*

Write 6 + 6 = 4 + 4 + 4.

Do they both equal 12? *(yes)*

Write 12 = 12 under 6 + 6 = 4 + 4 + 4.

Write "5 groups of 3" on the whiteboard.

Say: **With your cubes, arrange the cubes into 5 groups of 3.**

How many equal groups will you have? *(5)*

How many cubes are in each group? *(3)*

Build the equal-groups model along with the students. Check that each student's model is correct. Provide immediate corrective feedback when needed.

Say: **On your whiteboard write the *repeated addition* equation for this equal-groups sentence. What number is repeated?** *(3)*

How many 3s will you write? *(5)*

Read the *repeated addition* equation. *(3 + 3 + 3 + 3 + 3 = 15)*

3. Draw equal-groups models.

Use 1 number cube.

Say: **On your whiteboard next to the *repeated addition* equation draw the *equal-groups model* using circles and dots.**

How many groups or circles will you draw? *(5)* **Draw it.**

How many dots are in each group? *(3)* **Draw it.**

Check students' drawings of the equal-groups model. Draw the model on your whiteboard. Provide corrective feedback when needed.

Say: **Let's draw another *equal-groups model.* We are going to roll the number cube. The first number represents the number of groups. The second number represents how many in each group.**

Have a student roll the number cube.

Say: **What number did we land on?** *(answer will vary from 1-6)*

How many groups do we draw? *(answer will vary from 1-6)* **Draw it.**

Have a student roll the number cube again.

Say: **What number did we land on?** *(answer will vary from 1-6)*

How many in each group? *(answer will vary from 1-6)* **Draw it. How many in all?** *(answer will vary)*

Write the equal-groups sentence. *(answer will vary)* **Read it.**

Write the *repeated addition* equation for the equal-groups sentence. *(answers will vary)* **Read it.**

Practice — Time: 8 min

Activity 1: Students will work with a partner rolling a number cube to determine the number of groups and a repeated addition equation.

Have students turn to the *Practice Sheets* on pages 5 and 6. Distribute 1 number cube to each pair of math partners.

Say: **With your partner you will take turns rolling the number cube. The first roll will determine the number of groups. Write the number of groups on your sheet and then draw the groups in the space provided.**

The second roll will determine how many are in each group. Record how many in each group on your sheet and then draw dots in each group to represent the number in each group.

Finally, write a *repeated addition* equation for the equal-groups sentence. Use the picture you drew to check that your *repeated addition* equation is correct.

Allow students time to complete 3 problems. Provide corrective feedback when necessary. Have students share their equal-groups sentences and pictures.

Activity 2: Have students turn to the ***Practice Sheets*** on pages 7 and 8. Practice writing groups of sentences and repeated addition equations from an equal-groups model and vice-versa.

As students work ask question such as:

- How many equal-groups are in the model? *(answers will vary)*
- How many objects are in each group? *(answers will vary)*
- What number is repeated? *(answers will vary)*
- How many times is it repeated? *(answers will vary)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 3

Equal-Groups Model and Multiplication

Lesson Objectives	• The student will write multiplication equations from an equal-groups model. • The student will use multiplication terms when writing and reading multiplication equations and related models.	
Vocabulary	**Commutative Property of Multiplication:** a property of multiplication (but not division) that states that changing the order of the factors (numbers in the problem) will not change the product (the answer); for example, 5 x 8 = 40 and 8 x 5 = 40 **multiplication:** an operation that gives the total number when you put together equal groups	
Reviewed Vocabulary	equal, equal-groups, equal-groups model, repeated addition, equation, expression	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 20-31) • Whiteboard with marker • Counters (10)	• Student Booklet (pp. 11-16) • Whiteboard with marker • Counters (10 per student)

Preview

Say: **Today we will connect equal groups and repeated addition to write multiplication equations.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review the concept of "equals" by writing partner equations and repeated addition equations.

Write "2 + 2 + 2 = 5 + 1" on the whiteboard.

Say: **Read the problem.** *(2 + 2 + 2 = 5 + 1)* **Equal means the same as or the same amount on both sides of the equal sign. It does not mean answer. What does equal mean?** *(the same as on both sides of the equal sign)*

Look at the first expression. This is an example of repeated addition. If we were going to show this using counters, how many groups would there be? *(3)* **How many in each equal group?** *(2)*

What does 2 + 2 + 2 equal? *(6)*

Look at the second expression. Can I make 5 + 1 into an equal-group model? *(no)* **Why?** *(each group has to be the same amount)*

What does 5 + 1 equal? *(6)* **Is this equation correct? Is 2 + 2 + 2 the same as or equal to 5 + 1?** *(yes)*

Write "3 + 3 + 3 + 3 = 4 + 4" on the whiteboard.

Say: **Read the problem.** *(3 + 3 + 3 + 3 = 4 + 4)*

Are these expressions an example of equal groups? *(yes)* **How do you know?** *(same number in each group)* **How many counters would be in each group to show an equal-groups model in the first expression?** *(3)*

Look at the second expression, how many counters would be in each group to show an equal-groups model? *(4)*

Can we use skip counting to solve the first expression? *(yes)* **How many times are we counting by 3s to solve?** *(4)* **Count.** *(3, 6, 9, 12)* **What does 3 + 3 + 3 + 3 equal?** *(12)*

Look at the second expression. What is 4 + 4? *(8)*

Is 3 + 3 + 3 + 3 = 4 + 4 true? *(no)* **Why?** *(they do not equal the same amount)*

Modeled Practice **Time: 8 min**

1. Build equal-groups models. Write equal groups, repeated addition equations, and multiplication equations for each model.

   Give each student 10 counters and a whiteboard.

Say: **Is it possible to separate the 10 counters into 2 equal groups?** *(yes)* **How do you know?** *(because each group will get 5 counters; even number)*

Put these 10 counters into 2 equal groups and circle each group.

What is the equal-groups sentence for this model? *(2 groups of 5 equal 10)* **Write the equal-groups sentence on your whiteboard.**

What is the repeated addition equation for this model? *(5 + 5 = 10)* **Write the repeated addition equation on your whiteboard.**

Multiplication **is the operation used to find the total number when you put equal groups together.**

What is the operation called when we put equal groups together? *(multiplication)*

The equal-groups model is a model of repeated addition as well as a model of *multiplication*. *Multiplication* is the operation that gives the total number when you put equal groups together.

Can we use *multiplication* when we don't have equal groups? *(no)*

Can we use *multiplication* when we are taking away or losing some of the amount? *(no)*

When can we use *multiplication*? *(when putting together equal size groups)*

To write a *multiplication* equation for this equal-groups model, I will first count the number of groups. How many equal groups do I have? *(2)*

The first number in the *multiplication* equation will be 2.

Write 2 under the addition equation.

The second number for this *multiplication* equation will be the number of objects that are in each group. How many counters are in each group? *(5)*

Write 5 after the 2 on the whiteboard.

When we write a *multiplication* equation we use a symbol, the *multiplication* sign. The *multiplication* sign looks like this: ×. What is the symbol for *multiplication* I should use in this *multiplication* equation? *(×)*

Write the *multiplication* symbol in between the 2 and the 5 on the whiteboard.

How many counters does 2 × 5 equal? *(10)*

Is 5 × 2 the same as or equal to 10? *(yes)*

Therefore, 2 × 5 equals 10. Write = 10. Read the *multiplication* equation. *(2 × 5 = 10)*

2. Rearrange the counters to make 5 groups of 2 to write a repeated addition and a multiplication equation. Introduce the vocabulary term, Commutative Property of Multiplication.

   Have a student volunteer rearrange the counters into 5 groups of 2.

Say: **How many counters do we have in all?** *(10)*

Can we create 5 equal groups with the 10 counters? *(yes)* **Rearrange the counters into 5 equal groups. How many counters are in each equal group?** *(2)*

How many groups? *(5)* **What is the equal-groups sentence for this model?** *(5 groups of 2)* **Write it.**

What is the repeated addition expression for 5 groups of 2? *(2 + 2 + 2 + 2 + 2)* **Write 2 + 2 + 2 + 2 + 2 on your whiteboard.**

What is the *multiplication* expression that goes with the repeated addition expression? *(5 × 2)* **Write = 5 x 2 after the repeated addition.**

Do both sides equal 10? *(yes)*

Is 5 × 2 the same as 2 x 5? *(yes)*

Write 5 × 2 = 2 × 5. What do both sides equal? *(10)*

Why is this equation true? *(because both sides equal the same amount)*

In *multiplication* we can change the order in which the numbers are multiplied and it does not change the answer.

This is called the *Commutative Property of Multiplication*.

Write “commutative” on the whiteboard.

Say: **In the word commutative, we hear the word commute. What word?** *(commute)* **This means to move. What does it mean?** *(to move)* **This can help us to remember what commutative means.**

Moving the numbers around in a *multiplication* problem does not change the answer.

Write the expression 4 × 3 on your whiteboard. 4 × 3 is the same as what? *(3 × 4)* **Write = 3 × 4 after 4 × 3.**

What do we know about the *Commutative Property of Multiplication*? *(the order of the numbers can change and it does not change the answer)*

3. Have students write repeated addition and multiplication equations from equal groups sentences.

   Have students turn to the *Modeled Practice Sheet* in their Student Booklets.

Say: **Read the problem together. Ready, read: "Julian sees 4 baskets with 3 apples in each basket. He writes the *multiplication* expression 4 × 4 × 4 to find the total number of apples in all 4 baskets. Is his *multiplication* expression correct? Why?"**

First, let's draw the picture Julian saw. How many baskets or groups of apples? *(4)* **Draw 4 circles. How many apples in each basket?** *(3)* **Draw 3 dots in each circle.**

What is the equal-groups sentence? *(4 groups of 3)* **Write the equal groups sentence.**

What is the repeated addition expression for this model? *(3 + 3 + 3 + 3)* **Write it.**

What is the *multiplication* expression? *(4 × 3)* **Write it.**

Look at Julian's *multiplication* expression. Is it correct? *(no)* **Why?** *(he wrote the repeated addition expression instead of the multiplication expression)*

If Julian wrote 3 × 4, would this be correct? *(yes)* **Why?** *(you can change the order of the numbers in a multiplication expression)*

Practice — Time: 8 min

Activity 1: Have students turn to the *Practice Sheets* on pages 12 and 13. Students will work with a math partner.

Say: **With your math partner complete the practice sheets. Write a repeated addition equation and a *multiplication* equation for each equal groups model.**

Check for understanding with questions such as:

- What is the equal-groups sentence for this problem? *(answers will vary)*
- Which number is repeating? *(answers will vary)*
- If I add 1 more dot to this group, what would the new *multiplication* equation be? *(it would not be an equal groups model so you could not write a multiplication equation for it because all the groups are not equal)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 14. Students will work with a math partner.

Say: **With your math partner complete the practice sheet.**

Check for understanding with questions such as:

- Which number is repeating? *(answers will vary)*
- In the *multiplication* equation, what does the first number represent? *(the number of groups)*
- What would be the repeated addition equation for $6 \times 4 = 24$? *($4 + 4 + 4 + 4 + 4 + 4 = 24$)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
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Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 4

Modeling Multiplication Using the Number Line

Lesson Objectives	•The student will use the number line as a model of multiplication. •The student will apply the language of multiplication to the number line diagram.	
Vocabulary	**No new vocabulary words are introduced.**	
Reviewed Vocabulary	equal-groups model, equation, expression, multiplication, repeated addition	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 32-43) • Number line (laminated or in plastic sleeve) • Whiteboard with marker	• Student Booklet (pp. 17-22) • Number line (1 per student) • Whiteboard with marker

Preview

Say: **Today we will use the number line to model and solve multiplication problems.**

Engage Prior/Informal Knowledge **Time: 3 min**

Have students practice equal groups. Draw 3 groups of 6 using circles and dots on the whiteboard.

Ask questions such as:

- How many groups do I have? *(3)*
- How many are in each group? *(6)*
- What is the equal-groups sentence for this model? *(3 groups of 6)*
- What is the repeated addition equation for 3 groups of 6? *(6 + 6 + 6 = 18)*
- What is the multiplication equation for 6 + 6 + 6? *(3 × 6 = 18)*
- Is the answer the same for 3 × 6 and 6 × 3? *(yes)*
- How do you know they are the same? *(possible answers: both equal 18; 3 × 6 = 6 × 3; from the Commutative Property of Multiplication)*

Complete the problems below by drawing the equal groups on the whiteboard and using the questions above, if time permits:

- 7 groups of 3
- 4 groups of 7
- 9 groups of 4

Modeled Practice **Time: 8 min**

1. Demonstrate using the number line to solve multiplication problems. Review skip counting to reinforce equal groups.

   Give each student a whiteboard and a laminated number line or one in a plastic sleeve to be written on by a removable marker. Write "3 × 5 =" on the whiteboard.

Say: **Is multiplication related to addition or subtraction?** *(addition)* **How do you know?** *(multiplication is combining equal groups, so we can write a repeated addition sentence to solve)*

Which direction did you move along the number line? *(to the right)*

Write the multiplication equation on your board. Read this multiplication equation as an equal-groups sentence. *(3 groups of 5)* **Write the equal-groups sentence. Draw the picture for this equal-groups sentence.** *(allow time for students to draw)*

Look at your picture of 3 groups of 5. Which number is repeating in this problem? *(5)*

How many times is 5 repeating? *(3 times)*

When using the number line to solve addition and subtraction, we started at the greater number and then counted on or counted back.

When using the number line to solve multiplication problems, we start at 0 and make equal-sized jumps.

Where do I start on the number line to solve multiplication? *(0)*

Place your finger on 0 and have the students place their fingers on 0 on their number lines.

Say: **I need to show 3 groups of 5 or 3 jumps of 5. Think about when we used the hundreds chart to skip count. We made**

jumps to show the amount we were skip counting. Count 5 to find out where the first jump will land.

Darken the line segment between the hash marks on the number line as you count to 5.

Watch For	**Students may count the hash marks on a number line instead of the spaces between the numbers. For example, the number line begins with 0; the student may count the 0 hash mark as 1, not the length between the 0 and 1 as 1. To avoid this misconception have students darken and count the space between the numbers rather than the hash lines by the numbers.**

Say: **Where does my first set of 5 land?** *(on the 5)*

This represents 1 jump of 5.

Draw an arch over the first set of 5 and write "×1" above the arch. Have students match their number lines following the teacher's model.

Say: **How many jumps of 5 do I need to make?** *(3)*

I will count the next set of 5. Darken the line segment between each number as you count the next set of 5.

Where does my second set of 5 land? *(10)*

This represents 2 jumps of 5.

Draw an arch over the second set of 5 and write "×2" above the arch. Have students match their number lines following the teacher's model.

Say: **Instead of counting out 5 more I could also use my knowledge of skip counting to know where the next jump of 5 will land.**

If I jump 5 more what number will my jump land on? *(15)*

How do you know? *(skip count by 5s; count by 5s 3 times)*

Draw an arch over the third set of 5 and write “×3” above the arch. Have students match their number lines following the teacher’s model.

Say: **What number did I land on after 3 jumps of 5?** *(15)*

What is 3 × 5? *(15)*

How does this number line represent 3 × 5? *(it shows 3 jumps of 5 each)*

2. Students will work along with the teacher to practice representing multiplication on a number line.

Have students turn to the ***Modeled Practice Sheet*** in their Student Booklets. Clear off your number line so it may be used again.

Say: **Look at your practice sheet. Read the first multiplication equation.** *(4 × 6 =)*

Read the multiplication equation as an equal-groups sentence. *(4 groups of 6 equals 24)*

How many equal groups are we going to draw? *(4)* **Draw 4 circles. How many in each group?** *(6)* **Draw it.**

Place your pencil on the number line where you think we will start our jumps.

What is the size of each jump or each group? *(6)*

Check that students have placed their pencils on 0 on the number line. Provide immediate corrective feedback when necessary.

Say: **If we are going to make 4 jumps of 6, where will the first jump land?** *(on the 6)*

Draw an arch on the number line to represent your first jump of 6 starting at 0.

Label the first arch ×1 to represent 1 jump of 6.

Check students' work. Draw and write the same on your number line. Continue to demonstrate each step on your number line as students work on their *Modeled Practice Sheets*.

Say: **We need to jump 6 again. Where will you land on the number line?** *(12)*

If students do not know the second multiple of 6 have them darken the line segment between the hash marks on the number line to count out the next set of 6.

Say: **Draw an arch and label the arch ×2 to represent the second jump of 6 on your number line.**

How many jumps of 6 must we make to model this problem? *(4)*

On your number line count out 6 more to find where you will land for 3 jumps of 6. Double-check your jumps by darkening the line between each number as you count to 6.

Where did you land? *(18)*

Mark the third jump of 6 with an arch and label it ×3.

Mark the fourth jump of 6. Which number did you end on? *(24)*

What is 4 × 6? *(24)*

Check students' work. Provide immediate corrective feedback when necessary.

Practice — Time: 8 min

Activity 1: Students will work with a math partner to solve multiplication problems using the number line. Have students turn to the *Practice Sheet* on page 18.

Say: **Work with your math partner to represent the problem on the number line and find the answer. Draw arches and label each arch on the number line. Also write the multiplication problem**

as an equal-group sentence to remind yourself to think of the number of groups and the size of each group.

Ask questions such as:

- How many jumps will you make? *(answer will depend on the problem)*
- What number did you land on? *(answer will depend on the problem)*
- Did you need to count out the next set or did you know the sequence for the number counted? *(answer will vary)*

Activity 2: Have students turn to the ***Practice Sheets*** on pages 19 and 20. Students will work with a math partner. The students will fill in the missing information on the number line and solve the multiplication problem using the information from the number line.

Say: **With your math partner work on the next Practice Sheets. Fill in the boxes and use the number line to help you solve the multiplication problem.**

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module Multiplication & Division Relationships
Lesson 5

Strip Diagrams for Multiplication

Lesson Objectives	•The student will use the number line to create strip diagrams that represent multiplication. •The student will use correct mathematical language when describing the strip diagram and the part-whole relationship of multiplication.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	multiplication, unknown, expression, equation	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 44-55)	• Student Booklet (pp. 23-28)

Preview

Say: **Today we will use strip diagrams to represent and solve multiplication problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Review using a strip diagram to solve addition and subtraction problems.

Have students turn to the *Engaged Practice Sheet* in their Student Booklets.

Say: **Look at the strip diagram for problem #1. The top strip represents the whole. What does it represent?** *(the whole)* **Below the whole are parts. What is below the whole?** *(parts)*

Are 18 and 7 the parts or the whole in the strip diagram? *(the parts)*

The letter *n*, or the unknown, is what we are looking for. The letter *n* represents an unknown number. This unknown number is the whole in the part-part whole relationship.

If we have 2 parts and are looking to solve for the whole, what operation do you use, addition or subtraction? *(addition)*

Write an addition equation for this model on the line below the strip diagram. *(18 + 7 = 25)*

What is 18 + 7? *(25)*

Check students' work. Provide immediate corrective feedback when necessary.

Say: **Look at the strip diagram for problem #2.**

The letter *n* represents the unknown number. Is the unknown number a part or the whole? *(a part)*

What is the whole for this strip diagram? *(30)*

What part do you know? *(24)*

We know 1 part and the whole. What operation do you use, addition or subtraction, to solve for the other part? *(subtraction)*

Write a subtraction equation for this model on the line below the strip diagram. *(30 – 24 = 6)*

What is 30 – 24? *(6)*

If time allows, have students write a story problem for 1 of the strip diagram.

Possible writing prompts:

- You bake cookies and some are eaten.
- You earn money for helping your neighbor do two different tasks.
- The animal shelter receives new dogs or some dogs are adopted.

Modeled Practice — Time: 8 min

1. Connect the number line to the strip diagram by using both representations to solve the same problem.

   Use the *Modeled Practice Sheet*. Complete the sheet as the lesson progresses. Have students work along with you.

Say: **Read the multiplication problem.** *(3 × 6 =)*

Write and read the equal-groups sentence. *(3 groups of 6)*

How many arches will we need to draw on the number line? *(3)*

How many numbers are we jumping or skipping over? *(6)*

On the number line draw 3 groups of 6 to find the answer. The first group of 6 is already drawn for us. Draw an arch over the shade potion and write "×1" above it.

To find the second group of 6, we count over 6 spaces. What number did you land on for 2 groups of 6? *(12)* **Draw an arch and label it "×2."**

How many more groups do we need? *(1)*

How do you know? *(3 groups of 6)* **Count 6 more spaces, draw an arch and label it "×3."**

What number did you land on for 3 groups of 6? *(18)*

How many equal groups are there in this problem? *(3)*

What is the length of each equal group? *(6)*

What does 3 groups of 6 equal? *(18)*

Let's use this number line to draw a strip diagram for this same problem.

Draw a bracket, like the bottom of a rectangle, from 0 to 6. This represents the 1 equal part. Write "6" inside the bracket.

Draw a second bracket to represent the second jump of 6. Write "6" inside the bracket.

How much is 2 groups of 6? *(12)*

Make 1 more bracket along the bottom. Label it "6."

Is each part the same size, or length? *(yes)*

How much does each part represent? *(6)*

How many parts did we draw? *(3)*

What does 3 groups of 6 equal? *(18)* **Above the number line draw a long bracket from 0 to 18 to represent the whole or total.**

We know 3 × 6 equals 18 because we worked it out already so let's try 1 more that we have not solved.

2. Transition from using the number line to using only the strip diagram.

Walk students through the steps, continuing to show each step on the ***Modeled Practice Sheet*** as the lesson progresses. Have students work along with you.

Say: **Look at the strip diagram at the bottom of the page.**

How many parts are drawn? *(4)*

What is the value of each part? *(6)* **Write it in each part.**

Each part represents an equal group of 6. How many parts of 6 are in this model? *(4)*

What multiplication problem does this strip diagram represent? *(4 × 6)* **How do you know?** *(each group is 6 and there are 4 groups or parts)*

Write 4 × 6 under your diagram. Are we solving for the part or the whole? *(the whole)*

The whole is the unknown, represented by the letter *n*.

We multiply to find the unknown, just like we add the parts to find the whole.

Multiplication is similar or related to what operation? *(addition)*

What is the repeated addition expression for 4 × 6? *(4 + 4 + 4 + 4 + 4 + 4 or 6 + 6 + 6 + 6)*

What does 3 × 6 equal? *(18)*

Can we start at 18 and then count on 6 more to find the answer to 4 × 6? *(yes)*

Write 18 in the workspace area to the right side of your strip diagram.

We know that multiplication is repeated addition. What number are we adding repeatedly? *(6)*

In your workspace add 18 + 6. What does 18 + 6 equal? *(24)*

What does 4 × 6 equal? *(24)*

Is 24 the whole or the part? *(whole)*

Write "24" as your solution to this multiplication problem.

Practice

Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 25 and 26. Students will work with a math partner. Students will be asked to write a multiplication equation from a given strip diagram and draw a strip diagram from a given multiplication problem.

Say: **Read the first problem on your sheet. What is the question asking you to find?** *(how many hours of video games Trevor played for 3 days)*

How many hours did he play each day? *(4)* **How many days?** *(3)* **How can we solve?** *(multiplication)* **How do you know?** *(it is faster than adding 4 hours for 3 days)*

What is the multiplication problem we need to solve? *(4 × 3 =)*

Using the number line, what number do we begin? *(0)* **How many numbers will we jump?** *(3 or 4 depending on how the group solves)*

How many total jumps? *(3 or 4 depending on how the group solves)*

Shade and make the rectangle to show the parts under the jumps. What is 4 × 3 equal? *(12)*

What if we made [3 or 4] parts rather than [3 or 4]? Would the answer still be 12? *(yes)* **How do you know?** *(in multiplication you can solve with the numbers in any order)*

Work with your math partner to answer problems #2-5 on your Practice Sheets.

Check for understanding, asking such questions as:

- What is the value of each part? *(answers will vary depending on the problem)*
- How many equal-sized parts are needed for the model? *(answers will vary depending on the problem)*
- How did you solve this problem? *(repeated addition, skip counting, using facts I already know)*

Teacher Note
Students may switch the order of the factors when writing the multiplication sentence or when drawing the bar model. The Commutative Property of Multiplication applies so students will get the same answer either way it is drawn or written. If students draw or write it differently from one another, take this time to remind students of the Commutative Property of Multiplication and accept both methods as long as the correct answer is found.

Activity 2: Have students turn to the ***Practice Sheets*** on page 26. Students will write a story problem for 1 of the strip diagrams they have solved for question #6. Students may work in partners or on their own to write a story problem. Remind students for multiplication story problems there is a set amount that is being repeated multiple times, and the question is asking for the whole or total amount. Have students share their story problems with the group.

Possible story problem prompts:

- Washed cars and earned the same amount for each car
- Soda sold in a set amount (e.g., 6-pack)
- Read the same number of pages in a book over several days

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 6

Multiplication With Arrays

Lesson Objectives	• The student will use arrays as a model of multiplication. • The student will use mathematical language such as array and equal groups to describe models of multiplication.	
Vocabulary	**array**: a rectangular arrangement of objects in equal rows and equal columns **row:** Items arranged in a horizontal line **column:** Items arranged in a vertical line	
Reviewed Vocabulary	Commutative Property of Multiplication, equal groups, equation, expression, multiplication	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 56-65) • Examples of arrays in the classroom • Whiteboard with marker • Counters (20)	• Student Booklet (pp. 29-33) • Counters (16 per student) • Whiteboard with marker (1 per student)

Preview

Say: **Today we will work to solve multiplication problems using *arrays*.**

Engage Prior/Informal Knowledge — Time: 3 min

Review equal groups. Use 20 counters.

Say: **These counters represent crayons in a box. I would like to find the total number of crayons.**

Counting by 1s would take a while. Could I use equal groups to find the total number of crayons faster? *(yes)*

Let's make groups of 2.

Allow 1 student to demonstrate making equal-size groups to find the total.

Say: **Let's skip count by 2 since we put the counters in equal groups of 2: 2, 4, … 20.**

How many groups are there? *(10)*

How many are in each group? *(2)*

If I have 10 groups of 2 crayons, how many crayons do I have? *(20)*

There are 20 crayons.

Could we make equal groups of 5? *(yes)*

Allow 1 student to demonstrate making equal-size groups to find the total.

Say: **Let's skip count by 5 since we put the counters in equal groups of 5: 5, 10, 15, 20. There are 20 counters.**

How many groups are there? *(4)*

How many are in each group? *(5)*

If I have 4 groups of 5 crayons, how many crayons do I have? *(20)*

There are 20 crayons. Using equal groups of 2 or 5 helped us find the total number of crayons.

Making equal groups is useful to count objects quickly. Give some examples when you might use equal groups to find a total. *(possible answers: counting students in a class, cupcakes in a box, eggs in a carton, money, minutes on a clock, books sold at the book fair)*

Modeled Practice — Time: 8 min

1. Create an equal-groups model to transition multiplication represented in equal groups to multiplication represented in an array.

   Give each student 15 counters and the ***Modeled Practice Sheet***. Have students organize the counters into 3 groups of 5 on the table, circling each group.

Say: **Now let's put the counters into 3 equal groups and circle each group on your Modeled Practice Sheet.**

How many counters are in each group? *(5)* **How many total counters?** *(15)* **How do you know?** *(skip counted by 5s)*

What is the equal-groups sentence for this model? *(3 groups of 5 equal 15)* **Write it.**

What would be the repeated addition equation for this equal-groups model? *(5 + 5 + 5 = 15)* **Write it.**

Multiplication is the combining of equal groups. What is the multiplication equation for this equal-groups model? *($3 \times 5 = 15$)*

Can we write 5×3 and still have it equal 15? *(yes)* **Why?** *(in multiplication you can change the order of the numbers and not change the answer)* **Write both multiplication equations on the sheet.**

2. Use same counters to link equal-group models to an array.

Say: **Now I am going to use 15 counters but arrange them differently.**

Create a 3-row by 5-column array on the table or whiteboard.

Say: **This is an *array*. An *array* is a rectangular arrangement of objects in equal lines across and equal lines down.**

It has the same number of items in each *row* and the same number of items in each *column*.

A *row* runs across the page. Everyone run your finger across the table the same way the *row* runs across a page.

Let's count the number of *rows* in the *array*. Ready? Count 1, 2, 3. How many *rows*? *(3)* **Make an array on your sheet matching mine.**

A *column* runs up and down the page. Everyone run your finger up and down the table the same way the *columns* run up and down a page.

Let's count the number of *columns* in the *array*. Ready? Count 1, 2, … 5. How many *columns*? *(5)*

Point to the rows and then the columns when naming the array for the students.

Say: **This *array* is 3 by 5. Make a circle around each *row*. How many groups?** *(3)* **Are the groups equal?** *(yes)*

Did I add or take away any counters from the equal groups model? *(no)*

How many counters make up this 3 by 5 *array*? *(15)* **How do you know?** *(we skip counted the equal groups)*

We circled the *rows* in this *array*. There are 3 *rows* of 5. What is the equal groups sentence for this *array*? *(3 groups of 5 equal 15)* **Write it.**

What is the repeated addition equation for this *array*? *(5 + 5 + 5 = 15)* **Write it.**

What is the multiplication equation for this *array*? *(3 × 5 = 15 or 5 × 3 = 15)*

What changed from the equal-groups model to the *array* model? *(acceptable answers include the arrangement or the look of the model, if students say nothing press them to think of how it looked before compared to now)*

Did the quantity change from the equal-groups model to the *array* model? *(no, 3 groups of 5 equals 15 either way the counters are organized)*

Change the array to 5 rows and 3 columns.

Look at my *array*. How many *rows*? *(5)* **How many *columns*?** *(3)* **How many in all?** *(15)* **Does it matter which way we make the *array*?** *(no)* **Why?** *(the Commutative Property of Multiplication)*

It does not matter because we know that switching the rows and columns results in the same answer. We still have the same number of counters. This is the Commutative Property.

3. Students create an array and write mathematical sentences and equations for the array.

   Distribute 1 more counter to each student and a whiteboard. Students will write the equations for the array on their whiteboard.

Say: **With your counters create an *array* that has 2 *rows* and 8 *columns*.**

How many counters are in each *row*? *(8)* **Check your *array*, does each *row* have the same number of counters?** *(yes)*

How many counters in each *column*? *(2)* **Does each *column* have the same number of counters?** *(yes)*

What is the *array* you created? *(2 by 8 or 8 by 2)*

How many *rows* of 8? *(2)*

What is the repeated addition equation for 2 *rows* of 8? *(8 + 8 = 16 or 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16)*

Write the repeated addition equation on your whiteboard.

What is the multiplication equation for this repeated addition equation and *array*? *(2 × 8 = 16 or 8 × 2 = 16)*

Write the multiplication equation on your whiteboard.

Think about the equal-groups model and the *array* model. Turn to your math partner and discuss how these 2 models both represent multiplication.

Wait for students to discuss. Prompt discussion with sentence starters such as:

- The equal-groups model and the array both show equal groups and multiplication because…
- Multiplication is combining of equal groups, both models…
- The equal groups model represents multiplying by….
- The array model represents multiplying by…

Have each group share their explanation of how these models represent multiplication.

Practice — Time: 8 min

Activity 1: Students will work with math partners to create 2 more arrays using the same 16 counters.

Say: **Listen carefully to this story. Molly ordered cupcakes from the bakery. The cupcakes came in 1 box arranged in an *array*. Molly counted 4 *rows* of 4 cupcakes in the box. Can you make this cupcake *array* with your 16 counters?** *(yes)* **What must you remember about an *array* that is similar to the rule for equal**

groups? *(all rows have equal amounts and all columns have equal amounts)*

Once you and your partner make the original cupcake *array* with the counters, write the repeated addition and the multiplication equations for the *array* on your whiteboard.

Is there another way that the bakery could have made an *array* with the cupcakes? Work with a partner to make a second array with the 16 counters.

Monitor students' work closely. Provide corrective feedback when necessary. Students should make an 8 by 2 array and a 4 by 4 array with the 16 counters. If students try 3 or 5 rows, let the students evaluate for themselves if the arrangement works.

Possible questions or prompts to ask while students work:

- Do all of the rows or columns in your *array* have equal number of counters? *(should be yes; if no, prompt students to see how they can fix it)*
- Can you use the Commutative Property of Multiplication, or switch the order of the numbers being multiplied, to create a second *array*? *(yes, it creates an 8 by 2 array)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 30. Students will relate arrays with specific mathematical sentences. Provide corrective feedback when necessary.

Say: **Work with your math partner to complete the Practice Sheet.**

Ask the following questions when students have completed the sheet.

Say: **How many dots did you draw for 7 rows of 6?** *(42)*

What is the multiplication equation for this *array*? *(7 × 6 = 42)*

What are 2 equal-groups sentences for 5 × 2 = 10? *(5 groups of 2 equals 10 or 2 groups of 5 equals 10.)*

What is the repeated addition equation for the 4 by 3 *array* on your paper? *(3 + 3 + 3 + 3 = 12 or 4 + 4 + 4 = 12)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 7

Finding Area: A Model for Multiplication

Lesson Objectives	• The student will use array models to solve area problems. • The student will create an array to solve area problems. • The student will apply language related to multiplication and arrays to describe how to solve area problems.	
Vocabulary	**area:** the measure of how many square units the figure covers **square units:** a unit used to measure area. Square inches, square feet, and square meters are units used to measure area	
Reviewed Vocabulary	array, columns, Commutative Property of Multiplication, equal groups, equation, expression, multiplication, rows	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 66-77) • Square tiles (24)	• Student Booklet (pp. 34-39) • Square tiles (30 per student)

Preview

Say: **Today we will use arrays to solve *area* problems.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review arrays. Have students turn to the *Engaged Practice Sheet* in their Student Booklets.

Say: **Let's review what we know about arrays. Look at the array of beach balls.**

A row runs across the page. How many rows of beach balls are there? *(7)*

A column runs up and down the page. How many columns of beach balls are there? *(3)*

There are 7 rows and 3 columns of beach balls. We can say that there are 7 groups of 3 beach balls in each.

What is the repeated addition expression that represents this array? *(3 + 3 + 3 + 3 + 3 + 3 + 3)*

How many beach balls are there in all? *(21)*

What is the multiplication equation that represents this array? *(7 × 3 = 21)*

Rotate the *Engaged Practice Sheet* to portrait orientation.

Say: **Now I am going to rotate, or turn, the page to find another array using the same beach balls. Are there still 21 beach balls?** *(yes)*

We still have the same number of beach balls. It doesn't matter which way we start the array. You can start with the rows first or the columns first.

Why can we start with either the rows or the columns? *(Communicative Property of Multiplication; in multiplication you can* *change the order of the parts multiplied and still have the same amount or answer)*

How many rows are there now? *(3)*

How many columns are there now? *(7)*

There are 3 rows and 7 columns of beach balls or 3 groups of 7 beach balls in each.

What is the repeated addition expression that represents this array? *(7 + 7 + 7)*

What is the multiplication expression that represents this array? *(3 × 7)*

What is 3 × 7? *(21)*

We can see that 7 × 3 and 3 × 7 equal 21.

What do we know about the Commutative Property of Multiplication? *(states we can change the order of the numbers we multiply and still have the same answer)*

Knowing that changing the order of the numbers multiplied will result in the same answer is going to be helpful when we find *area*.

Modeled Practice — Time: 8 min

1. Students will find area using an array of tiles. Distribute the square tiles to the students.

   Point to the floor in the room.

Say: **Look at the floor in this room. An architect used *area* to design the floor space of this room. *Area* is the measure of how many *square units* the figure covers. Think about a room with tile floors. The tiles are like *square units*. A *square unit* is a unit of measurement for *area*.**

Arrange 24 tiles in an array of 4 rows of 6 tiles.

Say: **Now let's use the tiles to make an array. Make 4 equal groups of 6.**

Check students' arrangement of tiles into an array.

Say: **These are tiles in my bathroom. We are going to find the *area* of my bathroom using an array.**

How many rows are there? *(4)*

How many columns are there? *(6)*

What multiplication problems represent this array of tiles? *(4 × 6 and 6 × 4)*

Rotate the array of tiles as you say the following statement.

Say: **Both 4 × 6 and 6 × 4 represent the array of tiles.**

Can you count the tiles to find the *area*? *(yes)*

Counting the *squares units* is one way to find area. Is there a faster way to count? *(yes)* **How?** *(accept reasonable answers such as groups of 2s)* **Now count the tiles.**

How many tiles are there in all? *(24)*

We know from counting the tiles that 4 × 6 and 6 × 4 equal 24.

What is the *area* of the bathroom floor? *(24 square units)*

24 tiles cover the bathroom floor.

Watch For	**Students may confuse area and perimeter. For example, students may count the distance around the figure when solving for area instead of how many squares units the figure covers. Explain to students the difference between area and perimeter by using square tiles or grid paper.**

Students will find area using a shaded array on grid paper. Use the *Modeled Practice Sheet*. Move the tiles aside.

Say: **Let's put away the tiles and find the *area* of the carpet in the bedroom on the sheet.**

The squares represent the *area* of the carpet in the bedroom. Now shade in the squares of the carpet.

Now let's outline the carpet with your pencil. As you outline, find the number of rows and columns.

How many rows? *(5)*

Write "5" to represent the number of rows.

How many columns? *(9)*

Write "9" to represent the number of columns.

What multiplication expressions represent the shaded *area*? *(5 × 9 or 9 × 5)*

Rotate the *Modeled Practice Sheet* to landscape orientation.

Say: **Both 5 × 9 and 9 × 5 represent the carpet or shaded *area*. Write the multiplication problems.**

What is the *area* of the carpet in the bedroom? *(45 square units)*

The *area* of the carpet in the bedroom is 45 *square units*.

Now let's check our work. Instead of counting each shaded square, let's skip count the shaded columns.

On our paper, circle each column to show the equal groups.

How many in each group or column? *(5)*

Can you skip count the equal groups? *(yes)*

How should we skip count? *(by 5s)*

Ready, count: 5, 10, … 45.

The answer is correct. The *area* of the carpet in the bedroom is 45 *square units*.

Practice **Time: 8 min**

Activity 1: Students will create arrays to solve multiplication problems. Have students turn to the *Practice Sheet* on page 36. Students will work with a math partner to complete the activity.

Say: **You and your math partner will work together using tiles to solve a multiplication problem. Record your work on your Practice Sheet.**

Monitor students' work and provide corrective feedback when necessary. After students have completed the activity take time to discuss their findings. Allow students to explain how they solved the problem.

Ask questions such as:

- How did you use the tiles to solve the problem? *(to build an array)*
- What 2 multiplication problems could you have used to find the area without having to count all the tiles one by one? *(10 × 3 or 3 × 10)*
- How many tiles were in each equal group? *(10 or 3 depending if they circle the rows or the columns)*
- How did you check your answer? *(possible answers include using repeated addition, skip counting by 10, counting the tiles one by one)*
- Which method do you think is better to find the area of a large space, using an array or counting each tile one by one? *(using an array, accept reasonable explanations)*

Activity 2: Have students turn to the *Practice Sheet* on page 37. Students will work in pairs to find the area of places in a neighborhood using grid paper.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 8

Modeling Multiplication

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">• The student will use equal-groups models, the number line, and array models to solve multiplication and area problems.
• The student will review language related to multiplication models to solve problems.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new vocabulary words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">area, array, columns, Commutative Property of Multiplication, equal groups, equation, expression, multiplication, rows</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 78-87)
• Whiteboard and marker</td><td>• Student Booklet (pp. 40-44)
• Whiteboard and marker (1 per student)</td></tr>
</table>

Preview

Say: **Today we will review the multiplication models we have been using to solve.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review equal groups. Use the whiteboard. Have students work on their whiteboards.

Say: **Let's review equal groups. Draw 7 groups of 3 using circles and dots.**

How many circles did you draw? *(7)*

How many dots are in each circle? *(3)*

Does each group have the same amount? *(yes)*

The equal-groups model is a model of repeated addition and multiplication. Multiplication is the operation that gives the total number when you put equal groups together.

Now write the repeated addition equation for this equal-groups model on your whiteboard.

What is the repeated addition equation? *(3 + 3 + 3 + 3 + 3 + 3 + 3 = 21)*

What is 3 + 3 + 3 + 3 + 3 + 3 + 3? *(21)*

Now write the multiplication expression. *(7 × 3 or 3 × 7)*

What is 7 × 3? *(21)*

Is 7 × 3 the same as 3 × 7? *(yes)*

Why are they the same? *(3 groups of 7 is the same as 7 groups of 3; the Commutative Property of Multiplication)*

3 groups of 7 is the same as 7 groups of 3, and both have the same answer of 21.

Modeled Practice Time: 8 min

1. Students will review solving multiplication problems using the number line.

   Have students turn to the *Modeled Practice Sheet* in their Student Booklets. The teacher and students will complete the steps together as the lesson progresses.

Say: **Read the first problem. Ready, read: “3 × 8. Solve 3 × 8 using the number line.”**

What is the equal groups sentence for 3 × 8? *(3 groups of 8 or 8 groups of 3)* **Let’s use 3 groups of 8 to solve.**

What is the size of each jump on the number line? *(8)*

How many jumps will we make? *(3)*

Why? *(you will make 3 jumps because there are 3 groups)*

We are going to make 3 jumps of 8.

Where do I start on the number line? *(0)*

Where will the first jump land? *(on the 8)*

Draw an arch on the number line to represent the first jump of 8 starting at 0. Write “×1” above the arch.

Say: **Draw an arch to represent your first jump on the number line starting at 0.**

Count 8 more. Where will the second jump land? *(on the 16)*

Draw a second arch from 8 to 16 on the number line and write “×2.”

Say: **Draw an arch to represent your second jump and write × 2 above the jump.**

Count 8 more from 16. Where will the third jump land? *(on the 24)*

Draw a third arch from 16 to 24 on the number line and write “×3.”

Say: **Draw an arch to represent your third jump and write × 3 above the jump.**

What is 3 × 8? *(24)*

3 × 8 or 3 equal groups of 8 equals 24.

2. Students will review solving multiplication problems using arrays.

   Complete the next problem on the *Modeled Practice Sheet*.

Say: **Next let’s use an array model to solve the multiplication problem. Here is an array of circles.**

A row runs across the page. How many rows of circle? *(4)*

A column runs up and down the page. How many columns? *(5)*

There are 4 rows of 5 circles. We can say that there are 4 groups of 5 circles.

What is the repeated addition expression that represents this array? *(5 + 5 + 5 + 5)* **Write it.**

What is the multiplication expression that represents this array? *(4 x 5)* **Write it.**

How many circles are there in all? *(20)*

What does 4 x 5 equal? *(20)*

3. Students will review solving area problems using arrays.

   Continue with the next problem on the *Modeled Practice Sheet*.

Say: **Now let’s find the area of the shaded figure. Area is the measure of how many square units the figure covers. What is area?** *(the amount of square units a figure covers)*

Outline the shaded area with your pencil. As you outline, find the number of rows and columns.

How many rows? *(2)*

Write "2" to represent the number of rows.

How many columns? *(8)*

Write "8" to represent the number of columns.

What multiplication problems represent the shaded area? *(2 × 8 or 8 × 2)*

Rotate the *Modeled Practice Sheet* to landscape orientation.

Say: **Both 2 × 8 and 8 × 2 represent the shaded area. Write the multiplication equations.**

What is the area of the shaded figure? *(16 square units)*

How did you solve 2 × 8? *(counted the squares, added 8 + 8)*

The area of the shaded figure is 16 square units.

Instead of counting each shaded square to check our answer, let's skip count the shaded columns.

How should we skip count? *(by 2s)*

Let's count by 2s. Ready, count: 2, 4 …16.

The answer is correct. The area of the shaded figure is 16 square units.

Practice **Time: 8 min**

Activity 1: Students will create arrays to solve multiplication problems. Have students turn to the *Practice Sheet* on page 41. Students will work with a math partner to complete the activity.

Say: **You and your math partner will work together using different models to solve multiplication problems. Record your work on your sheets.**

Monitor students' work and provide corrective feedback when necessary. After students have completed the activity, take time to discuss their findings. Allow students to explain how they solved the problem.

Ask questions such as:

- How did you use the number line to solve the problem? *(answers will vary)*
- What 2 multiplication problems could you have used to find the area without having to count all the tiles one by one? *(4 × 7 or 7 × 4)*
- How did you check your answer? *(possible answers include using repeated addition, skip counting, counting the squares one by one)*

Activity 2: Have students turn to the *Practice Sheet* on page 42. Students will review the multiplication models in a small group activity. Tell students to first read the multiplication problem on the sheet and write the repeated addition equation. Then students will pass or rotate their booklets. Next, students will draw an array using circles before rotating again. Then students will shade the array on grid paper and then rotate. Finally, the students will solve the multiplication problem.

Teacher Note
Accept an array of 7 rows and 6 columns and an area model of 7 rows and 6 columns.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6minutes

Module: Multiplication & Division Relationships
Lesson 9

The Multiplication Table

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will use the multiplication table as a tool for solving unknown multiplication facts.
• The student will use mathematical language related to multiplication, such as factor, product, and multiple.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">multiplication table: a numerical form of an array that shows a multiplication or a division problem
factor: the numbers multiplied together to find a product; for example, 3 (factor) × 6 (factor) = 18 (product)
product: the answer to a multiplication problem.
multiple: repeated groups of the same amount; the product of any two whole numbers; for example, 8 is a multiple of 4 because it is the product of 4 and the whole number 2</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">column, equation, multiplication, row,</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 88-95)
• Multiplication table
• Whiteboard with marker</td><td>• Student Booklet (pp. 45-48)
• 2 different colored counters (15 per student)
• Multiplication table (1 per student)
• Number cubes (2 per student pair; 10 sided recommended)</td></tr>
</table>

Preview

Say: **Today we will work with a *multiplication table*. This math tool is designed to help us solve multiplication and division problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Have a discussion about math "tools." Discuss the word tool, naming some household tools, e.g., hammer, wrench, saw.

Say: **Tools are used to make tasks easier. What are some tools you use at home?** *(allow a variety of answers such as hammer, wrench, screwdriver)* **We also use tools in math. A math tool is used to solve math problems.**

Brainstorm with the students some math tools and how the tools are used to help solve problems. If possible, have a few math tool examples out for students to see.

Ask questions such as:

- How does this tool help us? When do you use this tool?
- Have you used this tool before? In what type of situation?

Possible tools to discuss:

- Calculator – to find solutions to difficult calculation problems
- Ruler – to determine the length of an object
- Hundreds chart – to solve mathematical problems
- Number line – to solve mathematical problems
- Counters – arrange in various ways to help solve problems
- Base-10 materials or place value chips – to help solve problems in the hundreds

Modeled Practice **Time: 8 min**

1. Use the ***multiplication table*** to discuss factors and products.

   Distribute a ***multiplication table*** to each student.

Say: **A *multiplication table* is a tool that can be used to solve multiplication and division problems. The numbers are in an array.**

Place your finger on the top corner, on the 1. Run your finger along the top row, going across the page.

The top row on the *multiplication table* is a list of *factors*. *Factors* are numbers multiplied together to get the answer. Name a *factor* in the top row. *(allow a variety of answers, 1 – 10)*

For example, in the multiplication equation 3 × 4 = 12, 3 and 4 are the *factors*.

What are *factors* in a multiplication equation? *(the numbers being multiplied together)*

In the multiplication equation 6 × 7 = 42, which 2 numbers are the *factors*? *(6 and 7)*

Since there are 2 *factors* in a multiplication fact there are 2 places to find *factors* on the *multiplication table*. Where is one place to find factors? *(top row)*

The second list of *factors* is in the far left column, going down.

Place your finger on the top corner, on the 1 again. Run your finger down the far left column. This column is another list of *factors*.

When multiplying 2 *factors* together the answer is called the *product*. The *product* is the answer to a multiplication equation.

In the equation 3 × 4 = 12, which number would be the *product*? *(12)*

What is the math word for the answer to a multiplication equation? *(product)*

On the whiteboard in large letters write "Factor × Factor = Product." Leave this on the whiteboard for the rest of the lesson.

Say: **Look at the *multiplication table* in front of you. If the *factors* are the top row and the far left column, which numbers are the *products*?** *(the numbers in the middle or all the rest of the numbers)*

2. Students will solve multiplication problems using the multiplication table.

Have a *multiplication table* displayed for the students. Work along with students on this multiplication table as students work on theirs. Check that students have accurately followed directions throughout the lesson.

Say: **Place your finger on the 2 in the top row. Is 2 a *factor* or a *product*?** *(a factor)*

Place another finger on the 2 in the far left column. Is this 2 a *factor* or a *product*? *(a factor)*

What is the multiplication expression for these 2 factors? *(2 × 2)*

Run the top finger down and the side finger across the page until they meet on a number.

What number did our fingers meet on? *(4)*

Is 4 a *factor* or a *product*? *(product)*

What is the complete multiplication equation you just used the *multiplication table* to solve? *(2 × 2 = 4)*

Use the *multiplication table* to solve 6 × 8.

What *factors* will you place your fingers on? *(6 and 8)*

Run your fingers down the column and across the row until they meet. What number do your fingers meet on? *(48)*

What is 6 × 8? *(48)* **Do we get a different answer if we start at 6 in the top row and 8 in the column?** *(no)* **What if I put my finger on 8 in the top row and 6 in the column, what is the answer?** *(48)*

What is the mathematical word for 48, the answer to a multiplication problem? *(the product)*

Think back to our previous lessons. We used hundred charts and number lines to list *multiples*. A *multiple* is repeated groups of the same amount, like skip counting groups. What is a *multiple*? *(repeated groups of the same amount)*

Look down the column or across the row of 6, what are the *multiples* of 6? *(12, 18, … 60)*

Practice

Time: 8 min

Activity 1: Students will work with a math partner to find the products to multiplication facts on the multiplication table. Have students turn to the *Practice Sheet* on page 45. Each pair of students should use a multiplication table to solve.

Say: **You and your partner will use the *multiplication table* as a tool to solve unknown multiplication problems.**

Ask such questions as:

- What are the factors in this problem? *(answers will vary)*
- What is the mathematical word for the answer in a multiplication problem? *(product)*
- What is the product? *(answers will vary)*
- The Commutative Property of Multiplication states we can switch the order of the facts but the product stays the same. What is the other multiplication problem with these same factors?

Activity 2: Students will play *Connect Four* using a ***multiplication table***, 2 number cubes, and colored counters. Have students turn to the ***Practice Sheet*** on page 46. Each pair should play on one of their sheets. Distribute 2 different sets of 15 colored counters to each pair of students. Students will roll a number cube to determine who will go first. Then player 1 rolls both number cubes to make a multiplication problem. Player 1 solves the problem using the multiplication table, and then places the counter in the product. The next player does the same. If the product has already been marked, the student can first check if switching the order of the factors on the hundreds chart (move each factor from rows to columns and columns to rows) will provide a product that is not marked. If both products for the pair of factors have been marked with a counter then that player passes. The first player with 4 products in a row (horizontally, vertically, or diagonally) wins.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 10

Equal Share Division

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will model equal share division using counters.
• The student will demonstrate understanding of equal sharing by finding how many in each group when a set amount is shared equally.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">equal share: the breaking of quantities apart so that everyone gets the same amount; the resulting amount of the action of equal sharing
leftover: The amount unable to be divided into an equal group</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">equal groups</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 96-103)
• Counters (30)</td><td>• Student Booklet (pp. 49-52)
• Counters (30 per student pair)
• Set of Equal Share Charade cards (1 set for the group)</td></tr>
</table>

Preview

Say: **Today we will learn about division by sharing equal groups of objects among a number of people.**

Engage Prior/Informal Knowledge Time: 3 min

Display 10 counters broken into a group of 8 and a group of 2.

Say: **How many counters do you see in all?** *(10)*

Point to the group of two counters.

Say: **I asked my friend to put these counters into equal groups. She gave me this group of 2 and herself the group of 8.**

What are equal groups? *(same number in each group)* **Are these groups equal?** *(no)*

Did we both get the same amount of counters? *(no)* **Could I write a multiplication problem from these groups?** *(no)* **Why?** *(not equal)*

What might be a better way to share the counters so that we each get the same amount? *(give each person 5 counters)*

What if there were 5 friends, is there another way to equally share or place these 10 counters into equal groups? *(yes)* **What is one way to equally share 10 counters with 5 friends?** *(5 groups of 2 or 10 groups of 1)*

Place 15 – 20 more counters on the table. Have students place the counters into equal groups. Ask the following questions:

- How many groups?
- How many in each group?
- How do you know the groups are equal?

Modeled Practice **Time: 8 min**

1. Students will equally share counters among the members of the group. Discuss equal sharing, or breaking up an amount into equal groups.

   Place 15 counters (for 5 students) on the table for the group to divide (or 12 if there are 3 or 4 students in the group).

Say: **I would like to share or divide these counters equally among the group.**

When we break apart some quantity so that everyone gets the same amount, it is called an *equal share*. What is an *equal share*? *(breaking apart some quantity so that everyone gets the same amount)*

When there are no more left to share equally, we have completed an *equal share*. How do we know when we are done sharing? *(when there are not enough left to share equally)*

Your group has discovered a treasure of coins. Each person must get the same amount, or equal amounts.

Have a student volunteer pass out 1 counter to each student.

Say: **Each of you received 1 coin. Are there enough coins in the treasure to continue to equally share?** *(yes)*

Give each person 1 more counter. How many coins do you each have now? *(2)*

Are there enough for us to continue to share equally? *(yes)*

Give each person 1 more counter.

Say: **How many counters, or coins, do you have now?** *(3)*

How do we know when we are done? *(when nothing is left to share equally)*

How many coins did you each receive when we shared the amount equally? *(3 if started with 15, 3 or 4 if started with 12)*

Does everyone have the same amount? *(yes)* **Is this an *equal share*?** *(yes)*

Sometimes when we equally share there are *leftover* counters, or ones not able to be in a group. Were there any *leftover*? *(no)*

2. Students will equally share counters among the group with an amount that will result with a leftover counter.

   Have students return their counters to 1 pile. Place 1 more counter to make 16 (for 5 students) on the table for the group to divide (or 1 more to make 13 if there are 3 or 4 students in the group).

Say: **Your group has discovered another treasure chest of coins. You must share the coins equally.**

How many coins are in the treasure chest? *(a student volunteer may count to find the number of coins, 16 or 13)*

How many students are we going to equally share with? *(the number of students currently at the table)*

Have a student volunteer pass out 1 counter to each student.

Say: **Each of you received 1 coin. Are there enough coins in the treasure to continue to make an *equal share*?** *(yes)*

What does equally share mean? *(everyone gets the same amount)*

Give each person 1 more coin. How many coins do you each have now? *(2)*

Are there enough for us to continue to equally share? *(yes)*

Give each person 1 more counter.

Say: **How many counters, or coins, do you have now?** *(3)*

Are there enough for us to continue to equally share? *(no)*

Why not? *(there is only 1 counter left)*

Would you each have *equal shares* if one of you were to receive the extra coin? *(no)*

There is 1 *leftover* coin that cannot be equally shared among all of you. This coin is the *leftover*. How many coins are *leftover*? *(1)*

Are we done sharing the coins equally? *(yes)*

How do we know when we are done? *(when nothing is left to share equally)*

How many coins did you each receive when we shared the amount equally? *(3 if started with 15, 3 or 4 if started with 12)*

How many *leftover* coins are in the treasure chest? *(1)*

3. From the problem just worked, fill in the blanks on the ***Modeled Practice Display*** as the students supply the information.

Say: **We will now record what we modeled with our counters.**

First we start with the number of coins we found in the treasure chest. How many coins did we try to equally share? *(16 or 13)*

Next we record the number of people who equally shared the coins. How many students received an *equal share*? *(the number of students in your group)*

Then we record the number of coins each person received after all the coins were shared equally. How many did each student receive? *(3 or 4)*

Finally, we record the number of coins that were *leftover* after the coins were shared equally. How many coins are *leftover*? *(1)*

Have a student volunteer read the equal sharing sentence: "16 shared equally with 5 students equals 3 per student with 1 leftover."

Say: **What does *equal share* mean?** *(everyone gets the same amount)*

Why might you have some *leftover*? *(there are not enough counters to share equally)*

Practice Time: 8 min

Activity 1: Students will work with a math partner using counters to solve equal sharing problems.

Have students turn to the ***Practice Sheets*** on pages 50 and 51. Distribute 30 counters per each set of math partners.

Say: **With your math partner read the equal sharing problem on your Practice Sheet. Then use the counters to solve the problem. Record your work by filling in the equal sharing sentence.**

Ask such questions as:

- How many counters did you need to start with? *(answers vary depending on problem number)*
- How many equal shares are you going to make? *(answers vary depending on problem number)*
- What do you write when nothing is leftover? *(0)*
- When there is some leftover, why can't everyone get more? *(because there is not enough for each person to get another, everyone must get an equal or same amount)*

Activity 2: The students will play *Equal Share Charades* with their math partner. Place the ***Equal Share Charade cards*** in 1 stack facedown in the middle of the table so everyone can reach.

The first student will draw an *Equal Share Charade* card from the stack in the middle. Using only counters, the student will act out the equal share sentence on the card. The student may not say a word to his or her partner as he or she works. The partner much watch carefully, counting and recording as the other student silently demonstrations the problem. Once the student has completely solved the problem, the partner watching reveals the equal group sentence he or she thinks is being solved. If correct, each student receives a point. If incorrect, the student with the counters must demonstrate it again to help his or her partner get the correct answer.

Then, the partners switch roles for the next problem. Students may continue to work out as many problems possible in the time available.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication and Division Relationships
Lesson 11

Equal Share Division Using Strip Diagram

Lesson Objectives	• The student will model equal share division using the strip diagram. • The student will use language related to division to explain partitioning into equal groups.	
Vocabulary	**division:** an operation that tells how many equal groups there are or how many are in each group; for example, 12 ÷ 3 = 4	
Reviewed Vocabulary	equal groups, equal share, equation, expression, leftover, strip diagram	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 104-113) • *Leftover Situation* cards • Whiteboard with marker • Counters (30)	• Student Booklet (pp. 53-57) • Counters (20 per student pair) • Whiteboard with marker (1 per student)

Preview

Say: **Today we will be using strip diagrams to solve equal share problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Use counters to review equal sharing and discuss leftover situations using the ***Leftover Situation cards***. Have a student volunteer draw a card and read it to the group. Allow students to use the counters to solve the problem on the card. Finally, discuss what should be done with the leftovers.

Possible questions to ask students:

- What should happen to the extra 2 students?
- What would you do with the extra dollar?
- Will someone get more treasure than the others?
- What would you do with the extra pizza slice?

Modeled Practice — Time: 8 min

1. Connect the models to division. Introduce division as a vocabulary term.

   Give each student a whiteboard and a marker. Have students share 21 equally with 3.

Say: **We have 21 carrot sticks to equally share between 3 people. How many people do we have to equally share the carrots with?** *(3)*

Draw 3 circles to represent the people. We will draw dots to represent the 21 carrots. We want to find out how many carrot sticks each person gets.

What is the best way to share the carrots equally? *(give each person 1 carrot at a time)*

Share each carrot or dot. Put a dot in each circle, counting to 21. When do we know we are done sharing? *(when the carrots or dots cannot be shared equally anymore)*

Allow time for the students to equally share 21 dots into 3 groups.

Say: **Are all the carrot sticks shared equally?** *(yes)*

Do we have any leftover carrots? *(no)*

How many carrots did each person get? *(7)*

Write "21 shared equally with 3 people equals 7 carrots per person with 0 leftover" on the whiteboard.

Write "division" on the whiteboard.

Say: ***Division*** **is the operation that tells how many equal groups there are or how many are in each group.**

In this example, we started with 21 dots and then divided the dots into equal groups. We equally shared the carrots into 3 groups. What is the math word that means breaking objects into equal groups? *(divide)*

2. Divide using a strip diagram.

Have students turn to ***Modeled Practice Sheet.*** The teacher and students will complete the sheet together as the lesson progresses.

Say: **We are going to *divide* the whole amount or the dots again. This time the dots represent fruit cups. Each person has to have an equal share. What does equal share mean?** *(everyone has the same amount)*

How do we know when we are done dividing the fruit cups? *(when there are no more left to equally share)* **Look at the strip diagram, how many total fruit cups or dots are we dividing?** *(24)*

24 is the whole, the amount we are dividing.

Let's divide the 24 fruit cups between the same 3 people. How many circles do we need to draw to represent the people sharing the fruit cups? *(3)* **Divide the bottom strip on the strip diagram into 3 equal-sized sections.**

What is our unknown, the whole or a part? *(a part)*

What is the whole? How many fruit cups are we starting with or dividing? *(24)*

How many people or groups are we dividing the fruit cups into? *(3)*

How many sections have we divided the bottom strip into? *(3)* **Why did we divide the bottom strip into 3?** *(it is the amount of people the fruit cups are going to be shared between)*

We are solving the *division* problem: 24 fruit cups divided by 3 people.

Equally share 24 dots. Place and count dots in each group until you have 24 dots shared between the 3 people. How do we know when we are done equally sharing? *(there are no more left to equally share)*

How many fruit cups or dots are in each group? *(8 dots or fruit cups each)*

In the strip diagram what is 1 part of the bottom section? *(8)*

Write "8" in each section of the bottom strip.

Are there any leftover? *(no)*

What is 24 divided equally into 3 groups? *(8)*

We can also see the relationship between *division* and multiplication here. 3 groups of 8 equals 24. We know that 3 × 8 or 8 + 8 + 8 = 24.

Look at the strip diagram. What are the 3 numbers in the division equation? *(24, 3 and 8)*

What would be the multiplication equation for this model? *(8 × 3 = 24 or 3 × 8 = 24)*

Fill in the blanks at the bottom of the sheet.

Keep 24 on the first line and then write 8 above the 3 and 3 above the 8 as you read the next section.

Say: **What if we made 8 groups and shared the 24 dots? How many would be in each group?** *(3)* **How do you know?** *(it is a number family, related to multiplication)* **Can we write 24 divided equally into 8 groups is 3 with 0 leftover?** *(yes)* **Why?** *(can change the order of the groups)*

Practice **Time: 8 min**

Activity 1: Have students turn to the ***Practice Sheet*** on page 54. Students will use counters and draw pictures to solve division problems.

Say: **On your sheet work with your math partner to solve the division problems. Draw dots to show how many you are equally sharing in each part on the strip diagram.**

Ask question and give prompts such as:

- How many are you starting with? *(answers vary depending on the problem)*
- How many sections will you divide the bottom strip into? How do you know? *(answers vary depending on the problem)*
- Show me with your counters how many each section received.
- How many dots did you draw in each section? *(answers vary depending on the problem)*
- What is the equal groups division equation for this problem? *(answers vary depending on the problem)*

Activity 2: Have students turn to the *Practice Sheet* on page 55. Students will work with their math partner to answer problems #4 and #5. Students will determine if an answer is reasonable or not. Students will need to discuss if the answer given looks correct, explain why or why not. Encourage students to draw a picture to help support their answer.

Have students share some of their findings once everyone has completed the problems.

Watch For	**Students may incorrectly divide objects into unequal groups. For example, students may pass out items into unequal groups with the counters or on paper. Demonstrate with counters how to pass out items and keep track as you put items into equal groups. Count the items in each group to check there is an equal amount.**

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 12

Division of Equal Groups

Lesson Objectives	•The student will use equal-groups models when dividing to find out how many groups. •The student will apply the meaning of partitioning by describing how to divide into equal groups.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	division, equal groups, equal share, equation, expression, leftover	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 114-119) • Counters (40)	• Student Booklet (pp. 58-60) • Counters (24 per student pair)

Preview

Say: **Today we will work again with division to find how many groups we can make when we share the same amount.**

Engage Prior/Informal Knowledge **Time: 3 min**

Use counters to review equal sharing using the equal-groups model. Place 32 counters on the table in 8 equal groups.

Say: **Let's review equal sharing using equal groups. We learned that when we equally share each person gets the same amount.**

Ask questions such as:

- How many groups of counters are on the table? *(8)*
- Without counting all the counters 1-by-1, how can we figure out how many counters in all? *(repeated addition, skip counting)*
- If there are 32 counters in all, how many are in each group? *(4)*
- What is the equal-groups sentence for this model? *(8 groups of 4 equals 32)*
- What is the repeated addition expression? *(8 + 8 + 8 + 8)*
- What are the 2 multiplication equations for this equal groups model? *($4 \times 8 = 32$ and $8 \times 4 = 32$)*

Remove 2 of the counters and arrange into 5 groups of 6.

Ask questions such as:

- If I removed 2 counters, how many counters do I have now? *(30)*
- How many are in each group? *(6)*
- How many groups did I make? *(5)*

- What is the equal-groups sentence for this model? *(5 groups of 6 equals 30)*
- What are the 2 multiplication equations for this model? *(5 × 6 = 30 and 6 × 5 = 30)*

Modeled Practice — Time: 8 min

1. Demonstrate division of equal shares through repeated subtraction.

   Give each student a whiteboard and a marker. Direct students to copy the teacher following instructions. Place 40 counters on the table in 1 group. Begin to place each counter in its own group.

Say: **There are 40 counters in this 1 group. If I make equal groups of 1 counter per group, how many groups would I have?** *(40)*

Write "40 groups of 1 equals 40" on the whiteboard. Stop making groups of 1 and place all counters back into 1 group of 40.

Say: **Make your board match mine. Read the division sentence together. Ready, read: "40 groups of 1 equals 40."**

Counting by 2s, I will make equal groups of 2 using all 40 counters. Predict how many equal groups do you think 40 counters in groups of 2 will make. *(accept student predictions)*

Have a student volunteer help you divide the 40 counters into groups of 2. Then have another student volunteer count the number of groups.

Say: **How many groups of 2 equal 40?** *(20)*

What is the equal-groups sentence for this model? *(20 groups of 2 equals 40)* **Write it under the last equal-groups sentence on your whiteboard.**

2. Connect the models to division.

Say: **In both examples we started with 40 counters and then divided the counters into equal groups. We were looking for how many equal groups we could make. What is the math word that means breaking a quantity into equal groups?** *(division)*

For the model on the table, we found that 40 divided into equal groups of 2 gives us 20 equal groups.

Think about the dividing process. We started with 40 and moved 2 into a group. What operation, addition or subtraction, are we using to help us solve this division problem? *(subtraction)*

After moving 2 counters to their own group, how many counters were left to divide into groups? *(38)*

Watch For	**Students may be unsure if addition or subtraction occurred when dividing into equal groups. For example, when asked if 2 was added or subtracted each time, the student does not answer or just guesses. Demonstrate the division and repeated subtraction again. Group the 40 counters and take away or subtract 2 counters at a time. Ask the students to explain what you are doing with the counters.**

Put the 40 counters back into 1 group. Have a student volunteer to help with the division of the counters for next problems.

Say: **This time divide the 40 counters into equal groups of 4. If each group will have 4 counters, what number will we be subtracting from the whole, 40, each time?** *(4)*

Multiplication is related to addition because it is the combining of equal groups. Division is related to subtraction because it is breaking apart into equal groups.

Teacher Note
It may be helpful to physically show equal groups being combined for multiplication and being broken apart for division.

Have the student volunteer subtract 4 counters from the original 40.

Say: **We have subtracted 4 counters from 40. How many counters are left?** *(36)*

We have 1 group of 4. Subtract another 4. How many counters do we have left? *(32)*

How many groups of 4 have we subtracted so far? *(2)*

Have the student volunteer continue to subtract 4 counters at a time, placing each group of 4 in its own pile on the table. Have the students continue to subtract as the volunteer moves the counters into equal groups of 4.

Once the volunteer is down to the last pile ask:

Say: **How do you know when you are done dividing counters into equal groups?** *(when there are no more left to share equally)* **We are down to only 4 counters left, can we make any more groups of 4?** *(yes, this is the last one)*

Do we have any counters leftover? *(no)*

40 was divided into how many groups of groups of 4? *(students should count the groups on the table and say 10)* **What is the equal groups sentence for 40 divided into groups of 4?** *(10 groups of 4 equals 40)*

On the whiteboard write "10 groups of 4 equals 40" under the other 2 sentences.

Say: **Write the equals-groups sentence on your board.**

These equal-groups sentences are written in the same way we wrote it to match our equal groups in multiplication. Now we are going to write it in a way to show that we are dividing rather than combining equal groups.

What is our whole number? How many did we start with? *(40)*

How many are in each group? *(4)*

How many groups did we make? *(10)*

When writing the division sentence we start with the whole. 40 divided into equal groups of 4 equals 10.

Write on the whiteboard under the equal-groups sentence "40 divided into equal groups of 4 equals 10 groups."

3. Divide the same counters into groups of 8.

Place all the counters back together in a group of 40. Have a student volunteer help divide the counters into groups of 8.

Say: **The first time we divided the counters into groups of 1; how many groups were made?** *(40)*

Next when we divided the counters into groups of 2; how many groups were made? *(20)*

Next when we divided the counters into groups of 4; how many groups were made? *(10)*

What is happening with the amount of counters in each group? *(it is getting bigger, it is increasing, it is doubling each time)*

What is happening to the total number of groups? *(it is getting smaller, it is decreasing, it is half as many as before)*

This time divide the counters into groups of 8 and make a prediction of how many groups you think we will make. *(accept student predictions and have students explain their predictions)*

Have the student volunteer subtract 8 counters at a time. Students can again subtract 8 from the total each time a new group is made. Once there is 1 group left ask questions such as:

- Can we make 1 more group of 8? *(yes, this is the last one)*
- Are there any leftover? *(no)*
- How many groups of 8 were we able to make from 40 counters? *(5)*

- Was your prediction correct or close? *(answer may vary)*

Write the division sentence “40 divided into equal groups of 8 equals 5” on the whiteboard.

Practice **Time: 8 min**

Activity 1: Students will work with a math partner to divide counters into equal groups. Distribute 20 counters to each student pair. Have students turn to the *Practice Sheet* on page 58.

Say: **With your math partner use the counters to answer the questions on your sheet.**

Activity 2: Have students turn to the *Practice Sheet* on page 59. Students will answer follow-up questions on their own from Activity 1. Students will need to explain and write about what happened when the counters were divided.

Watch For

Students may struggle solving problems with a remainder. For example, students may have difficulty dividing 20 counters into groups of 8 and place the remaining counters into a group or may dismiss the remaining counters altogether. Review equal sharing. Tell students that counters unable to be shared equally should be recorded as a leftover. The remaining counters cannot be placed in a group because the groups would no longer be equally sized. Emphasize the word equal and that it means the same amount.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships
Lesson 13

Practice with Division of Equal Groups

Lesson Objectives	• The student will use a strip diagram and repeated subtraction to solve division problems. • The student will complete equal group division sentences using a strip diagram.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	array, division, equal groups, equation, expression, strip diagram	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 120-137 • Counters (30)	• Student Booklet (pp. 61-69) • Counters (18 per student)

Preview

Say: **Today we will use the strip diagram to solve division problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Review using a strip diagram to solve multiplication problems using the *Engaged Practice Sheet.*

Say: **This is a strip diagram. We have used this before to solve multiplication problems. What is the multiplication problem?** *(4 × 9)* **What are we trying to find?** *(the answer, the whole, or the product)*

Write a question mark in the top strip because we are trying to find the product or the whole. How do we divide the bottom strip, or into how many parts? *(4 or 9)* **How many are in each part?** *(4 or 9, other factor than answered in previous question)*

What are the factors in this problem? *(4 and 9)* **What is the repeated addition expression to help us solve?** *(4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 or 9 + 9 + 9 + 9)*

What is a faster, more efficient way to find the answer than repeated addition? *(skip counting)*

What is 4 × 9? *(36)* **What is the mathematical word for the answer or the whole number?** *(product)* **Write 36 in the whole.**

Modeled Practice — Time: 8 min

1. Demonstrate dividing to find the number of groups using the strip diagram and counters.

   Use *Modeled Practice Sheet #1*. Place 30 counters on the table. Have a student volunteer count the counters on the table to find the total.

Say: **How many counters are on the table?** *(30)*

We have 30 counters to divide. What should we write in the whole? *(30)* **Write 30 in the whole.**

Divide the counters into groups of 5.

Say: **We need to divide the 30 counters into parts. We skip count by 5s well, so let's put the counters into groups of 5.**

We want to find out how many groups of 5 can be made with 30 counters. I know how much I am starting with, and I know the equal amount in each group. What do we not know, or what are we looking for? *(the number of groups)*

Since we do not know the number of groups, we cannot cut the bottom strip into equal parts or groups just yet. We only know that we are going to divide 30 into equal groups of 5.

Write "5" in the far left side of the bottom strip, leaving room to write 5 five more times. Draw a line after the first 5 to represent 1 group of 5. Instruct students to copy the teacher's model following instruction. Have a student volunteer move 5 counters from the original 30 to a separate pile.

Say: **We have made 1 group of 5. Do you have enough counters to make a second group of 5?** *(yes)*

I need to know how many counters I have left on the table before I divide again. In my workspace off to the side I will find out how many I have left.

When dividing are we combining or taking away? *(taking away)*

What mathematical operation do we use to take away? *(subtraction)*

What would be the subtraction problem for what just happened with the counters? *(30 – 5)* **Write the subtraction problem in your workspace and solve. What is 30 – 5? How many counters are left?** *(25)*

In the workspace write "30 – 5" then ask students what would be the answer. If students do not know, have a student volunteer count the remaining counters in the original pile.

Say: **We have 25 counters left. Can we take out another group of 5?** *(yes)*

Have a student volunteer move 5 counters to a separate pile. Write another "5" on the bottom strip of the strip diagram and draw a line to section it off from the remaining amount. Instruct students to copy the teacher's model.

Say: **We have subtracted 2 groups of 5 from 30. Can we continue to make more groups of 5?** *(yes)* **Use the workspace to show that we have taken away or divided to make another group of 5.**

How many counters are left in the original pile? *(20)*

While you ask this question subtract another 5 from 25 in the workspace to continue to show the repeated subtraction of 5.

Continue to have the students take out 5 at a time. Write "5" each time in the bottom section of the strip diagram and show the work in the workspace. Instruct students to copy the teacher's model. When there are only 5 counters left, ask these questions.

Say: **How many are left in the original pile?** *(5)*

Is that enough to make another group of 5? *(yes, the last one)*

How many groups of 5 did we make? *(6)*

Does the strip diagram show 6 groups of 5? *(yes)*

Do we have any leftover? *(no)*

Does our work in the workspace show that after subtracting 5 six times we are left with 0? *(yes)* **Have we equally shared or divided all 30 counters?** *(yes)*

2. Students will work along with teacher to complete the second example.

Have students turn to the ***Modeled Practice Sheet #2***. Distribute 18 counters to each student. The teacher and students will complete the steps together as the lesson progresses.

Say: **We are starting with 18 counters. On the strip diagram, where will we write the number 18?** *(in the top strip or whole)*

Write "18" in the whole of the model.

We are going to divide these 18 counters into groups of 3.

Have a student take out 3 counters from the 18 to make a separate pile. On the strip diagram, write "3" on the far left side of the bottom strip. Draw a line after it to represent 1 group of 3. Have students do the same on their paper.

Say: **We have made 1 group of 3. Do we have enough counters to make another group of 3?** *(yes)*

Use the workspace to show the division into an equal groups of 3. How many counters are we dividing into equal groups? *(18)*

How many counters will be in each group? *(3)*

In the workspace write "18 – 3 = 15." Have students do the same.

Continue to have student volunteers take out 3 counters at a time. Record the new group of 3 on the strip diagram. Determine how many counters are left using the workspace and repeated subtraction.

Say: **How many groups of 3 are in 18?** *(6)*

How many groups did the bottom strip in your strip diagram get broken into? *(6)*

How many times did you subtract 3 from 18 until you had 0 left? *(6)*

Explain how dividing the counters into groups, drawing a strip diagram, and working a repeated subtraction problem can all represent the same problem. *(accept answers where students demonstrate an understanding that in each situation a given amount is being broken up into equal groups)*

We have used the strip diagram to solve both multiplication and division problems. How is the strip diagram used

differently to solve these types of problems? *(accept a variety of answers, such as we are given the whole number and have to find the groups)*

Practice Time: 8 min

Activity 1: Students will each draw a strip diagram and solve using repeated subtraction. Then students will share their solution with the group.

Say: **I will give each of you your own separate division problem. You are to use the strip diagram and repeated subtraction to solve.**

When everyone has had a chance to solve their problem you will then share your work with the group.

Have students turn to the *Practice Sheet* on page 64. Provide a problem to each student. Allow time for students to complete their work. Provide teacher assistance when necessary and/or a hundreds chart to assist in repeated subtraction.

Examples include:

- 24 divided into groups of 6 equals how many equal groups? *(4 equal groups)*
- 32 divided into groups of 4 equals how many equal groups? *(8 equal groups)*
- 36 divided into groups of 6 equals how many equal groups? *(6 equal groups)*
- 27 divided into groups of 9 equals how many equal groups? *(3 equal groups)*
- 28 divided into groups of 4 equals how many equal groups? *(7 equal groups)*

Activity 2: Have students turn to the *Practice Sheets* on pages 65 and 66. Students will use a strip diagram to solve a division problem and complete the equal-groups division sentence for each completed strip diagram.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 14

Number Families for Multiplication and Division

Lesson Objectives	• The student will derive multiplication equations and division equations from a given number family. • The student will use the term number family to list the related equations in the number family.	
Vocabulary	**Dividend:** in division, the number that is being divided or the whole; for example, in 35 ÷ 5 = 7, 35 is the dividend **Divisor:** in division, the number that divides another number (the dividend); for example, in 35 ÷ 7 = 5, 7 is the divisor **Number family:** a set of related addition and subtraction equations, or multiplication and division equations, made from the same numbers; for example, 5 × 7 = 35, 7 × 5 = 35, 35 ÷ 7 = 5, 35 ÷ 5 = 7. **Quotient:** the result of dividing one number by another number; for example, in 15 ÷ 5 = 3, 3 is the quotient	
Reviewed Vocabulary	columns, Commutative Property of Multiplication, factor, equal-groups, equation, expression, multiplication, product, rows	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 138-147) • Counters (24) • *Multiplication/Division Number Family Go Fish* cards	• Student Booklet (pp. 70-74) • Index cards (3 per student) • Counters (24 per student)

Preview

Say: **Today we will use *number families* to see the connection between multiplication equations and division equations.**

Engage Prior/Informal Knowledge — Time: 3 min

Review equals groups to write a multiplication equation and a division equation. Introduce division vocabulary using the *Engaged Practice Sheet*.

Say: **Look at the array. How many rows?** *(6)* **How many columns?** *(6)* **What is the equal-groups sentence for this array?** *(6 groups of 6 equals 36)*

We have written an equal-groups sentence for both multiplication, the joining of groups, and division, the separating of groups. What is the multiplication equation for the array? *($6 \times 6 = 36$)* **What is the mathematical word for the answer to a multiplication equation?** *(product)* **What is one factor in this problem?** *(6)*

We are going to use the array and the equal-groups sentence to write a division equation. In division we start with the whole. How many dots in all? *(36)* **Write 36 on the first line. The mathematical word for the amount divided or the whole is the *dividend*. What is the name of the total amount being divided?** *(dividend)*

How many groups are in the array? *(6)* **We divided the *dividend* into groups of 6. The mathematical word for the amount of groups is the *divisor*. What is the word that tells us how many groups to divide the *dividend* into?** *(divisor)* **What is the *divisor*?** *(6)* **Write it.**

The answer in a division problem is called the *quotient*. What is the answer in a division problem called? *(quotient)* **What is the *quotient* in this problem?** *(6)* **Write it.**

Modeled Practice **Time: 8 min**

1. The students will use counters to create arrays before writing multiplication and division equations to represent each array.

   Distribute 28 counters to each pair of students.

Say: **With your counters create an array of 7 rows with 4 counters in each row.**

Create the array with the students. Check students' arrays. Provide immediate corrective feedback when necessary.

Say: **7 rows of 4 can be written as a multiplication problem. What is the multiplication expression related to this array?** *(7 × 4)*

How can you find the total number of counters in this array? *(accept answers where students are using previously learned strategies to solve unknown multiplication facts, e.g., repeated addition or splitting the array into smaller arrays)*

What is the multiplication equation for this array? *(7 × 4 = 28)*

What is another way to write 7 groups of 4? *(4 × 7 = 28)*

What do we know about the Commutative Property of Multiplication? *(accept reasonable answers such as we can switch the order of the factors but the product stays the same)*

2. Introduce the term number family. Use the previous examples to illustrate the new vocabulary word.

Say: **What 3 numbers are in this array?** *(7, 4 and 28)*

These 3 numbers, 7, 4, and 28, are a set of numbers that make up 4 different equations, 2 multiplication and 2 division equations. This set of equations is called a *number family*.

Only division equations that can be equally shared with no leftovers are part of a *number family*.

A *number family* is a way to describe a set of related multiplication and division equations made up of the same numbers with no leftover. What is a *number family*? *(has 4 equations, has 2 multiplication equations and 2 division equations)*

Point to the number family triangle on the *Modeled Practice Sheet*. Fill in the blanks as you continue the lesson.

Say: **This is a *number family* triangle to help illustrate how these numbers are related.**

28, 4, and 7 go in each corner of the triangle. We can use the numbers around the triangle to make multiplication and division equations. Write the 2 factors in the lower corners. Write the product in the upper corner.

Place your finger on 4 and move it around the triangle as you say the equation. Repeat with each equation.

Say: **4 times 7 equals 28. Write it.**

We can now write the corresponding multiplication equation. Change the order of the factors. What is the corresponding multiplication equation? *(7 times 4 equals 28)* **Write it.**

28 divided by 7 equals 4. Write it.

There is not a corresponding equation for division because we can only start with the whole. Why can't we write a corresponding division equation? *(we can only start with the whole)* **28 divided by 4 equals 7. Write it.**

However, when using the *number family* triangle you must be careful. Not every direction will give you a correct number sentence. Does 28 times 7 equal 4? *(no)*

Does 7 divided by 28 equal 4? *(no)*

We need to remember how we use the greatest number, the whole. In multiplication we combine groups or 2 parts to find

the product. In division we begin with the whole and then separate it into groups or parts.

What is the mathematical name for 28 in a multiplication equation? *(product)*

What is the mathematical name for 28 in the division equation? *(dividend)*

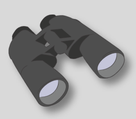
Can 4 be the product when using these 3 numbers? *(no)*

What is the mathematical name for 4 in a multiplication equation? *(a factor)*

What is the mathematical name for 4 in the division problem 28 divided by 4? *(the divisor)*

What is the mathematical name for 4 in the division equation 28 divided by 7 equals 4? *(the quotient)*

Watch For



Students may struggle with the vocabulary words as well as the order of numbers in the equations. For example, a student may state that 6 divided by 24 does equal 4. Refer back to the equations written on the Engaged Practice Sheet. Use the counters to illustrate each of these equations. Ask questions such as, if I have 6 counters can I make 24 rows of 4?

Practice **Time: 8 min**

Activity 1: The group will write multiplication and division equations in a round robin setting.

Have students turn to the *Practice Sheet* on page 72. In the number family triangle on each student's page, write in a different number family. The students are to write 1 multiplication equation for that number family in the top left hand box before passing the paper to the person to their left. Next, the students write a second multiplication equation for the number family that is now in front of them in the top right hand box. Students should continue passing papers to the left. Students will each have a new number family to write an equation for. Each student should have the opportunity to write 2 multiplication equations and 2 division equations. When the chart is complete have students pass back their papers to the original owner.

While students are writing the equations ask questions such as:

- Which number would be the product for your number family? *(answers will vary depending on the number family)*
- Which numbers are the factors for your number family? *(answers will vary depending on the number family)*
- Which number would have to be the dividend in the division equation for your number family? *(answers will vary depending on the number family)*
- Which numbers could either be the divisor or the quotient for your number family? *(answers will vary depending on the number family)*

Activity 2: Have the students play *Multiplication/Division Number Family Go Fish* as a group. Use the ***Multiplication/Division Number Family Go Fish cards***. Shuffle the cards and deal 7 cards to each player. Place the remaining cards in a pile facedown in the middle of the group. Students are trying to make a number family book by collecting 2 multiplication and 2 division equations for each number family set. Students take turns asking a single person if they have any equations for the number family of interest (insert the 3 numbers in the set here). For example, if the student is holding the card $3 \times 7 = 21$, he or she would ask, "John, do you have any equations for the number family 3, 7, and 21?" If yes, the player hands over the cards. If no, the player responds, "Go fish," meaning the asking student must draw a card from the pile in the middle of the group. Play continues until time runs out or until all number families have been collected.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 15

Multiplying by 0 and 1

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will multiply factors by 0 and 1.
• The student will apply the Zero Property of Multiplication and Identity Property of Multiplication to solve multiplication facts with 0 and 1.
• The student will apply language related to multiplication and the properties to explain their solutions.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">Zero Property of Multiplication: the product of 0 multiplied by any other factor is 0; for example, 32 × 0 = 0

Identity Property of Multiplication: the product of 1 multiplied by any other factor always equals that factor; for example, 32 × 1 = 32</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">array, Commutative Property of Multiplication, division, equal groups, equation, expression, factor, multiplication, number family, product, repeated addition</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 148-157)
• Whiteboard with marker</td><td>• Student Booklet (pp. 75-79)
• Whiteboard with marker (1 per student)</td></tr>
</table>

Preview

Say: **Today we will use our knowledge of multiplication models to multiply by 0 and by 1. We will also write division equations using number families.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review equal groups. Use the whiteboard. Have students work on their whiteboards.

Say: **Let's review equal groups. Draw 6 groups of 1 using circles and dots.**

Wait 10-15 seconds for students to draw the equal groups model. Draw the model on the whiteboard.

Say: **How many circles did you draw?** *(6)*

How many dots are in each circle? *(1)*

Does each group have the same amount? *(yes)*

There are 6 groups of 1 in each group.

Now write the repeated addition expression for this equal-groups model on your whiteboard.

Wait 3-5 seconds for students to write the repeated addition expression.

Say: **What is the repeated addition expression?** *(1 + 1 + 1 + 1 + 1 + 1)*

What is 1 + 1 + 1 + 1 + 1 +1? *(6)*

Now write the multiplication expression. *(6 × 1)*

Next draw the array that represents 6 × 1 on your whiteboard using squares.

Wait 3-5 seconds for students to draw the array.

Say: **What is 6 × 1?** *(6)*

Now rotate your whiteboard and look at the array. Is 6 × 1 the same as 1 × 6? *(yes)*

Why are they the same? *(6 groups of 1 is the same as 1 group of 6, there are the same number of squares when the array is rotated; it is the Commutative Property of Multiplication)*

6 × 1 is the same as 1 × 6 and both have the same product of 6.

Modeled Practice Time: 8 min

1. Students will solve multiplication problems using the Zero Property of Multiplication. Students will continue using the whiteboard. Write "7 × 0 =" on the whiteboard.

Say: **Read the problem.** *(7 × 0)*

Is 7 × 0 the same as 7 groups of zero? *(yes)*

Draw 7 groups of zero using circles and dots.

Wait 3-5 seconds for students to draw the equal groups.

Say: **How many circles did you draw?** *(7)*

How many dots are in each group? *(0 or nothing)*

There are 0 dots in each group. What is the repeated addition expression that represents the drawing? Write it. *(0 + 0 + 0 + 0 + 0 + 0 + 0)*

What is 7 × 0? *(0)*

Now add another circle, or group, to your drawing.

How many groups are there now? *(8)*

How many dots? *(0)*

We now have 8 groups of 0.

What is 8 × 0? *(0)*

Add another circle, or group, to your drawing.

How many groups? *(9)*

How many dots? *(0)*

What is 9 × 0? *(0)*

Do you notice a pattern? *(yes)*

What is the pattern? *(the answer or product is always zero)*

The rule can be explained by the *Zero Property of Multiplication.* The property states that a number multiplied by 0 is 0.

What is the *Zero Property of Multiplication*? *(a number multiplied by 0 is 0)*

Now let's use the *Zero Property of Multiplication* to solve some problems.

What is 0 × 1? *(0)*

What is 10 × 0? *(0)*

What is 0 × 100? *(0)*

What is 1,000 × 0? *(0)*

Now write your own problem using the *Zero Property of Multiplication* on your whiteboard.

Allow students to share their example. Put the whiteboards aside.

2. Students will solve multiplication problems using the Identity Property of Multiplication. Use the *Modeled Practice Sheet*.

Say: **Read the word problem together. Ready, read: "There are 12 desks and 1 pencil on each desk. How many pencils are there?"**

What is the equal-groups sentence for this problem? *(12 groups of 1)* **Write it.**

What is the multiplication expression for 12 groups of 1? *(12 x 1)* **Write it.**

Let's use the number line to solve 12 x 1.

What is the size of each jump? *(1)*

Each jump is 1. How many jumps will we make? *(12)*

We are going to make 12 jumps of 1.

Where do I start on the number line? *(0)*

Make 12 jumps of 1 on your number line starting at 0.

Wait 5-10 seconds for students to work.

Say: **Where did you land?** *(on 12)*

What is 12 × 1? *(12)* **Write it.**

How many pencils are there? *(12)*

What if there are 13 desks and 1 pencil on each desk. How many pencils now? Use the number line to solve.

Wait 3-5 seconds for students to work.

Say: **How many more jumps did you make?** *(1)*

What is the multiplication expression? *(13 x 1)*

What is 13 × 1? *(13)* **Write it.**

How many pencils? *(13)*

If I had 20 desks with 1 pencil on each desk, then how many pencils? Try to solve it in your head. *(20 pencils)*

Do you notice a pattern when multiplying by 1? *(yes)*

What is the pattern? *(a number multiplied by 1 is itself)*

The rule can be explained by the *Identity Property of Multiplication.* The property states that a number multiplied by 1 is itself.

Now let's use the *Identity Property of Multiplication* to solve some problems.

What is 1 × 1? *(1)*

What is 50 × 1? *(50)*

What is 1 × 200? *(200)*

What is 2,000 × 1? *(2,000)*

Now write your own problem using the *Identity Property of Multiplication* on your sheet. Write the corresponding multiplication equation. Then write 2 division equations in this number family.

Allow students to share their example.

Practice Time: 8 min

Activity 1: Students will create arrays to solve multiplication problems. Have students turn to the *Practice Sheet* on page 76. Students will work with a math partner to complete the activity.

Say: **You and your math partner will work together to solve the multiplication problems. Record your work on your sheet.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Which problem used the *Zero Property of Multiplication*? *(problem #1)*
- Which problem used the *Identity Property of Multiplication*? *(problem #2)*

- Explain the *Zero Property of Multiplication* in your own words. *(answers will vary)*

- Explain the *Identity Property of Multiplication* in your own words. *(answers will vary)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 77. Students will fill in the first row of the multiplication table with the 1 fact. Students will refer to this multiplication table in the next lessons and complete the multiplication table as they learn more facts and strategies.

Ask questions such as:

- We could have a row with the factors of 0, but what are all the products? *(it would be all zeros across the row)*

- What do you notice about the first row you completed? *(the answers increase by 1, it shows the Identity Property of Multiplication)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 16

Multiplying by 2, 5, and 10

Lesson Objectives	• The student will multiply factors by 2, 5, and 10 and write equations using knowledge of number families. • The student will use skip counting to solve multiplication facts with a factor of 2, 5, and 10. • The student will verbalize steps in solving problems with a factor of 2, 5, and 10 to explain mathematical thinking.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	area, Commutative Property of Multiplication, equal groups, equation, expression, Identity Property of Multiplication, multiplication, number family, skip count, Zero Property of Multiplication	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 158-167) • Whiteboard with marker	• Student Booklet (pp. 80-84) • Whiteboard with marker (1 per student)

Preview

Say: **Today we will use our knowledge of skip counting to multiply by 2, 5, and 10.**

Engage Prior/Informal Knowledge Time: 3 min

Review multiplication properties. Use the whiteboard. Have students work on their whiteboards.

Say: **Let's first review the Zero Property of Multiplication. What is the Zero Property of Multiplication?** *(a number multiplied by 0 is 0)*

The property states that a number multiplied by 0 is 0.

Write your own problem using the Zero Property of Multiplication on your whiteboard.

Allow students to share their example.

Say: **Let's solve some problems together using the Zero Property of Multiplication.**

What is 4 × 0? *(0)*

What is 0 × 17? *(0)*

What is 28 × 0? *(0)*

What is 143 × 0? *(0)*

What is 0 × 3,000? *(0)*

Now let's review the Identity Property of Multiplication. What is the Identity Property of Multiplication? *(a number multiplied by 1 is itself)*

The property states that a number multiplied by 1 is itself.

Let's use the Identity Property of Multiplication to solve some problems.

What is 1 × 9? *(9)*

What is 39 × 1? *(39)*

What is 1 × 88? *(88)*

What is 543 × 1? *(543)*

What is 4,000 × 1? *(4,000)*

Now write 2 problems using the Identity Property of Multiplication on your whiteboard.

Allow students to share their examples.

Modeled Practice — Time: 8 min

1. Students will use skip counting to solve multiplication facts with 2.

   Distribute a whiteboard and marker to each student.

Say: **We are going to solve multiplication problems using our knowledge of skip counting.**

Draw an array of 6 rows of 2 circles in each row.

Write the multiplication expression that represents the array.

What is the multiplication expression? *(6 × 2)*

Circle each group of 2. How many groups are there? *(6)*

What can I skip count by to find the total number of circles? *(by 2)*

We are familiar with counting by 2s. To solve we skip count by 2s. How many times? *(6)*

Point to each group and count. Ready, count: 2, 4 … 12.

What is 6 × 2? *(12)*

What 3 numbers are in this number family? *(2, 6, 12)*

What is the corresponding multiplication equation? *(2 × 6 = 12)*

What is the whole number? *(12)*

What are the division equations for this number family? *(12 ÷ 6 = 2 and 12 ÷ 2 = 6)*

When you are multiplying a number by 2, you can solve the fact by skip counting by 2s.

Let's practice multiplying by 2.

What is the repeated addition expression for 8 × 2 or 8 groups of 2? *(2 + 2 + 2 + 2 + 2 + 2 + 2 + 2)*

What is 8 × 2? *(16)*

What is the repeated addition expression for 7 × 2 or 7 groups of 2? *(2 + 2 + 2 + 2 + 2 + 2 + 2)*

What is 7 × 2? *(14)*

Skip counting by 2s will help you solve multiplication facts with 2.

2. Students will use skip counting to solve multiplication facts with 5.

   Have the students turn to ***Modeled Practice Sheet #1***. The teacher and students will complete the steps together as the lesson progresses.

Say: **Look at the area model here. How many rows?** *(7)*

How many columns? *(5)*

Write the multiplication expression used to find the area of the shaded model. *(7 x 5 or 5 x 7)*

What can I skip count by to find the area or the total number of square units? *(by 5)*

You skip count by 5s. How many times? *(7)*

Point to each row and count by 5s. Ready, count: 5, 10 … 35.

What is area of the shaded model? *(35)* **Write it.**

Using your knowledge of number families, what is the corresponding multiplication equation? *($7 \times 5 = 35$)* **Write it.**

What three numbers are in this number family? *(5, 7 and 35)* **We need to start with the whole or the greatest number to divide. What is one division equation for this number family***?* *($35 \div 5 = 7$ or $35 \div 7 = 5$)*

When you are multiplying a number by 5, you can solve the fact by skip counting by 5s.

Let's practice multiplying by 5.

What is 5×3? How many times do we skip count by 5s? *(3)* *(15)*

What is 8×5? *(40)*

When you are multiplying a number by 5, what do you skip count by? *(by 5s)*

3. Students will use skip counting to solve multiplication facts with 10.

   Continue working as a group. Have students turn to the ***Modeled Practice Sheet #2.***

Say: **Can you skip count by 10s to 100?** *(yes)*

Ready, count: 10, 20 … 100.

Knowing how to skip count by 10s will help solve facts with 10.

Read the problem. *(10×2)*

We can think of 10×2 as two dimes, or two 10-cent coins. To find how much 2 dimes are worth, what can I skip count by? *(10s)*

You skip count by 10s. How many times? *(2)*

Point to each dime. Ready, count: 10, 20.

What is 10 × 2? *(20)*

2 dimes are worth 20 cents. What is the corresponding multiplication equation for this number family? *(2 × 10 = 20)* **Write it.**

What are the 2 division equations for this number family? *(20 ÷ 10 = 2 and 20 ÷ 2 = 10)* **Write it.**

When you are multiplying a number by 10, what do you skip count by? *(by 10s)*

Let's practice multiplying by 10.

What is 10 × 3? *(30)* **Write it.**

What is 7 × 10? *(70)* **Write it.**

Practice — Time: 8 min

Activity 1: Students will create arrays to solve multiplication problems. Have students turn to the ***Practice Sheet*** on page 82. Students will work with a math partner to complete the activity.

Say: **You and your math partner will work together to solve the multiplication problems. Record your work on your sheet.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Is skip counting useful? Why? *(answers will vary)*
- In the first problem, 11 × 2, how would you skip count to solve? *(by 2s)*
- In the second problem, 8 × 5, how would you skip count to solve? *(by 5s)*

- In the third problem, 10×6, how would you skip count to solve? *(by 10s)*

Activity 2: Students will fill in the second, fifth, and tenth rows on the multiplication table with the 2s, 5s and 10s facts. Have students turn to the ***Practice Sheets*** on page 77. Students will continue to refer to this multiplication table in the next lessons and complete the multiplication table as they learn more facts and strategies.

Ask questions such as:

- When we fill in the multiplication table, we are skip counting and writing the multiples. Name 2 multiples for 5. *(answers will vary)*
- Name 2 multiples for 10. *(answers will vary)*
- Name 2 multiples for 2. *(answers will vary)*
- When we fill in the multiplication table, we are writing the product of 2 factors. What factor and 4 make a product of 40? *(the factor 10, $10 \times 4 = 40$)*
- What factor and 9 make a product of 18? *(the factor 2, $2 \times 9 = 18$)*
- What factor and 10 make a product of 100? *(the factor 10, $10 \times 10 = 100$)*
- Can you use the multiplication table to find division facts? *(yes)*
- Explain how you can use the multiplication table to find division facts and give an example. *(on the multiplication table a product divided by a factor shows the quotient; for example, 10 divided by 2 equals 5)*
- What do you notice about the third row after you completed it? *(the products skip count by 2s)*
- What do you notice about the sixth row after you completed it? *(the products skip count by 5s)*
- What do you notice about the eleventh row after you completed it? *(the products skip count by 10s)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 17

Solving for 9 Facts

Lesson Objectives	• The student will multiply factors by 9 and write equations using knowledge of number families. • The student will use the Make Ten Subtract the Factor strategy to solve facts with a factor of 9. • The student will verbalize steps in solving problems with a factor of 9 to explain mathematical thinking.	
Vocabulary	**Make Ten Subtract the Factor:** a strategy in which the student uses the Distributive Property to find the product of facts with a factor of 9; for example, $9 \times 6 = 10 \times 6 - 6 = 54$	
Reviewed Vocabulary	array, dividend, division, divisor, equation, factor, multiplication, number family, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 168-179) • Counters (40)	• Student Booklet (pp. 85-90) • Hundreds chart (1 per student) • Whiteboard with marker (1 per student)

Preview

Say: **Today we will learn a strategy, or steps, to solve the 9 facts quickly. We will also use our knowledge of number families to write the 4 equations.**

Engage Prior/Informal Knowledge — Time: 3 min

Review decades and subtracting 9 from each ten. Give each student a reusable (laminated or in sheet protector) ***Hundreds chart,*** a whiteboard, and a marker.

Say: **Look at the hundreds chart and find 10. Skip count by 10s. Shade in the multiples of 10 as you count. What is 10 × 4?** *(40)* **What is 10 × 6?** *(60)* **Write a multiplication problem with 1 factor as 10. Give to your neighbor to solve.**

Allow a few moments for the students to write the answer. Check it and then have students erase the boards.

Say: **Write 10 – 4 on your whiteboard. What is 10 – 4?** *(6)* **Under this problem write 60 – 4. What is 60 – 4?** *(56)* **What do you notice about the 2 problems?** *(answers will vary but may include: there is a 6 in the ones column, similar because in both problems you are subtracting from 10 or a multiple of 10)*

Knowing how to make 10 is important and can help us subtract from the decade quickly. What is 10 – 8? *(2)* **What is the other subtraction sentence for this fact family?** *(10 – 2 = 8)* **What is 80 – 8?** *(72)* **What is 20 – 2?** *(18)* **Knowing how to quickly subtract from 10 or the multiples of 10 will help you use the 9s strategy.**

Teacher Note
Subtracting and making 10 can be a very difficult concept for students. Using concrete materials alongside the hundreds chart can assist in understanding the pattern. Students may also need to write the problems vertically to show the regrouping.

Modeled Practice **Time: 8 min**

1. The students will use counters to create arrays to show the 9s strategy. The students will then write the equations in the number family for each fact.

   Distribute 40 counters on the table. Have students turn to the *Modeled Practice Sheet #1*. The teacher and students will complete the steps together as the lesson progresses.

Say: **Read the problem** *(9 × 4)* **First let's build an array to show this problem.**

We just practiced subtracting from 10. What is 10 – 9? *(1)* **What 3 numbers are in this number family?** *(10, 9 and 1)* **What is the other subtraction equation?** *(10 – 1 = 9)* **We know that equal means the same as, so 9 is the same as 10 – 1. What is 9 the same as or equal to?** *(10 – 1)*

Look at the spaces under the 9. Write "10 – 1." We are going to think of 9 as 10 – 1. What is the other factor in the problem? *(4)* **Write "4" after 10 – 1.**

Look at the next row. We now have 2 multiplication problems. To separate the problems they are placed in parentheses. Read the first problem. *(10 × 4)* **Read the second problem.** *(1 × 4)*

We need to multiply both parts of 9 by the other factor.

First, let's solve using the counters. Add another group of 4 to our array. How many groups of 4 now? *(10)* **What is 10 × 4?** *(40)* **Write 40 on the line below 10 × 4.**

What is 1 × 4? *(4)* **Write it on the line below that problem. We thought of 9 as 10 and added another group to the array. Now we need to subtract the array or the group that shows 1 × 4. There are 40 cubes. Now take away 4. What is 40 – 4?** *(36)*

Do we have 36 cubes left in our array? *(yes)* **What is 9 × 4?** *(36)* **What are the 3 numbers in this number family?** *(9, 4, and 36)*

Write the corresponding multiplication equation and the 2 division equations for this number family.

2. Review the steps to solve facts with a factor of 9.

   Have students turn to ***Modeled Practice Sheet #2***. Continue working as a group.

Say: **The strategy we just used was *Make Ten Subtract the Factor.* This only works for facts with a multiple of 9 because the difference between 9 and 10 is 1. Should we use this strategy to solve 1 × 9?** *(no)* **Why not?** *(faster or more efficient to solve using the Identify Property)*

What about 9 × 2? *(no)* **Why not?** *(faster to skip count)* **What about 5 × 9 or 9 × 10?** *(no, faster to skip count)*

We already know how to solve 9 times 0, 1, 2, 5, and 10.

The *Make Ten Subtract the Factor* is a strategy we should use to solve 9 times 3, 4, 6, 7, 8, and 9. Give me an example of a multiplication problem where we could use the *Make Ten Subtract the Factor* strategy. *(9 × 3, 9 × 4, 9 × 6, 9 × 7, 9 × 8, 9 × 9)*

Read the next problem. *(9 × 6)* **Step 1 is to think of 9 as 10 – 1. What is step 1?** *(Think of 9 as 10 – 1)* **Write "10" on the first line.**

Step 2 is to multiply 10 times the other factor. What is step 2? *(multiply 10 times the other factor)* **What is the other factor in this problem?** *(6)* **Write "6" on the line. What is 10 × 6?** *(60)*

What is step 1 of the strategy? *(Think of 9 as 10 – 1)* **What is step 2?** *(multiply 10 to the other factor)*

Step 3 is to subtract the other factor. What is step 3? *(subtract the other factor)*

We subtract the other factor because 9 is 10 – 1. What is the other factor? *(6)* **Write it on the line below 60.**

What is 60 – 6? *(54)*

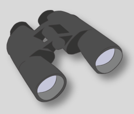
What is 9 × 6? *(54)*

What 3 numbers are in this number family? *(9, 6, 54)*

Write the corresponding multiplication equation and the 2 division equations. *(6 × 9 = 54, 54 ÷ 6 = 9, 54 ÷ 9 = 6)*

What is the strategy called? *(Make Ten Subtract the Factor)*

Watch For



Students may struggle with understanding why you subtract after you multiply. Build the problem with the array and show that 10 is like 1 more group, and to find the answer you have to subtract that group. The teacher can also show the problem using the Distributive Property with parentheses as in the first problem on the Modeled Practice Sheet.

Practice

Time: 8 min

Activity 1: Have students use the Make Ten Subtract the Factor strategy to solve problems. Have student identify the 3 numbers in the number family and then write the equations.

Have students turn to the ***Practice Sheets*** on pages 87 and 88. First students will write the words in the blanks for the strategy steps. Then have students complete the problems using the Make Ten Subtract the Factor Strategy.

While students are solving ask question such as:

- What is the other factor in the problem? *(answers will vary depending on the problem)*
- What is Step 1? Step 2? Step 3? *(think of 9 as 10 – 1, multiply 10 and the other factor, subtract the other factor)*

- What 3 numbers are in this fact family? *(answers will vary depending on the problem)*

Activity 2: Have students turn to the ***Practice Sheets*** on page 77. Students will fill in the ninth row on the multiplication table with the 9 facts. Students will continue to refer to this multiplication table in the next lessons and complete the multiplication table as they learn more facts and strategies.

Ask questions such as:

- When we fill in the multiplication table we are skip counting and writing the multiples. Name 2 multiples for 9. *(answers will vary)*
- When will fill in the multiplication table, we are writing the product of 2 factors. What 2 factors make a product of 36? *(9 and 4)*
- What 2 factors make a product of 81? *(9 and 9)*
- What 2 factors make a product of 72? *(9 and 8)*
- Can you use the multiplication table to find division facts? *(yes)*
- Explain how you can use the multiplication table to find division facts and give an example. *(on the multiplication table a product divided by a factor equals a factor, such as 10 divided by 2 equals 5)*
- What do you notice about the third row you completed? *(the products skip count by 2s)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 18

Solving for 6 Facts

Lesson Objectives	• The student will multiply factors by 6. • The student will use the Break-apart 6 strategy to solve facts with a factor of 6. • The student will verbalize steps in solving problems with a factor of 6 to explain mathematical thinking.	
Vocabulary	**Break-apart 6:** a strategy in which the student breaks apart the factor 6 into 1 and 5, then uses the Distributive Property to multiply 1 and 5 by the other factor, and add the products; for example, 6 x 7 = (5 x 7) + (1 x 7) = 42	
Reviewed Vocabulary	dividend, divisor, division, equation, expression, factor, multiplication, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 180-191) • Whiteboard with marker	• Student Booklet (pp. 91-96) • Whiteboard with marker (1 per student) • Counters (6 per student)

Preview

Say: **Today we will learn a strategy to solve the 6 facts quickly.**

Engage Prior/Informal Knowledge Time: 3 min

Review solving 5 and 1 facts. Give each student a whiteboard, marker, and 6 counters.

Say: **What is the best way to solve multiplication facts with a factor of 5?** *(skip count by 5s)*

Write 5 × 8 on your board. How many groups of 5 do you skip count? *(8)* **What is 5 × 8?** *(40)*

What is the Identity Property? *(any factor times 1 equals itself)*

Write 1 × 7 on your board. What is 1 × 7? *(7)*

In multiplication and addition we put groups or parts together to find the whole.

Review parts of 6.

Say: **When we add we have 2 parts or 2 groups. The parts do not have to be the same number when we add. For example, I can break apart 6 into 2 and 4. What are different ways to break apart 6?** *(1 and 5, 3 and 3, 4 and 2)*

Modeled Practice Time: 8 min

1. Students will solve multiplication facts using the Break-apart 6 strategy.

   Use *Modeled Practice Sheets #1* and *#2.*

Say: **Look at the array. Read the problem.** *(6 × 8)*

What are the 2 factors? *(6 and 8)*

Think about the different ways we broke apart 6. What facts do we know well? Think, do I know 8 × 1, and what about 8 × 4?

(allow a variety of answers but remind students that skip counting by 5s is fast and easily completed)

We know our 5 and 1 facts well so we will break apart 6 into 1 + 5. Look at the array. Draw a line separating 1 group of 6. How many groups of 6 are in the next array now? *(5)*

Write 1 and 5 in the boxes. We just created 2 new arrays and 2 new multiplication expressions for each array.

When solving facts with a factor of 6, step 1 is to break apart the 6 into 1 and 5.

What is step 1? *(break apart 6 into 1 and 5)*

Step 2 is multiply 1 and 5 by the other factor. What is step 2? *(multiply 1 and 5 by the other factor)*

What is the other factor in this problem? *(8)*

Write 8 on both lines.

What is 1 × 8? *(8)* **Write it below the problem.**

What is 5 × 8? *(40)* **Write it.**

Step 3 is add the products. What is step 3? *(add the products)*

What is 8 + 40? *(48)*

What is 6 × 8? *(48)* **What 3 numbers are in this number family?** *(6, 8, 48)* **What is the corresponding multiplication equation***? (8 × 6 = 48)* **Write it.**

What is the whole or the dividend in this number family? *(48)* **What are the division equations?** *(48 ÷ 7 = 6 or 42 ÷ 6 = 7)*

This is the *Break-apart 6* strategy. It can be used to solve 6 times 3, 4, 6, 7, and 8. Give me an example of a multiplication problem for which we could use the *Break-apart 6* strategy. *(6 × 3, 6 × 4, 6 × 6, 6 × 7, 6 × 8)*

Should we use this strategy to solve 1 × 6? *(no)* **Why?** *(faster to solve using the Identity Property)*

What about 6 × 2? *(no)* **Why?** *(faster to skip count by 2)*

What about 5 × 6? *(no, faster to skip count by 5)*

What about 6 × 10? *(no, faster to skip count by 10)*

What about 6 × 9? *(no, use the Make Ten Minus the Factor strategy)*

We already know how to solve some of the 6 facts.

Read the next problem. *(6 × 6)*

What are the 2 factors? *(6 and 6)*

Can we use the *Break-apart 6* strategy to solve this problem? *(yes)*

Why? *(you are multiplying by 6)*

What is step 1? *(break apart 6 into 1 and 5)* **Do we have to break apart both 6s?** *(no)* **We only break apart one 6.**

Write 1 and 5 in the boxes.

What is step 2? *(multiply 1 and 5 by the other factor)* **What is the other factor?** *(6)* **Write 6 on both lines.**

Multiply 1 and 5 by the other factor.

What is 1 × 6? *(6)* **Write it.**

What is 5 × 6? *(30)* **Write it.**

Step 3 is add the products. What is step 3? *(add the products)*

What is 6 + 30? *(36)*

What is 6 × 6? *(36)* **What 3 numbers are in this number family?** *(6, 6, and 36)* **What is the division equation for this number family?** *(36 ÷ 6 = 6)* **Write it.**

Watch For	**Students may forget to add the products together to solve the multiplication fact. For example, in the first problem 6 x 8 of the Modeled Practice, the students may write 40 or 8 as the answer instead of 48 (8 + 40). Practice the steps of the strategy again and emphasize the last step to add the products together. Show the problem using an array and have students push the pieces together to show that you have to add the products to find the whole amount of the array.**

Practice

Time: 8 min

Activity 1: Have students use the Break-apart 6 strategy to solve problems.

Have students turn to the ***Practice Sheets*** on pages 93-95. First students will write the words in the blanks for the strategy steps. Then have the students complete the rest of the problems using the Break-apart 6 strategy.

While students are solving ask questions such as:

- What is the other factor in the problem? *(answers will vary depending on the problem)*
- What is Step 1? Step 2? Step 3? *(break apart 6 into 1 and 5, multiply 1 and 5 by the other factor, add the products)*

Activity 2: Students will fill in the sixth row on the multiplication table with the 6 facts. Have students turn to the ***Practice Sheets*** on page 77.

Ask questions such as:

- Name 2 multiples for 6. *(answers will vary)*
- What 2 factors make a product of 18? (*6 and 3)*

- What 2 factors make a product of 42? *(6 and 7)*
- What is 36 divided by 6? *(6)*
- What other problems can we fill in? *(fill in the corresponding facts in the 3s column – 3 × 6, 7s column – 7 × 6, 8s column – 8 × 6)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheet*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 19

Solving for 4 and 8 Facts

Lesson Objectives	• The student will multiply factors by 4 or 8 and write equations using knowledge of number families. • The student will use the Double It strategy to solve facts with a factor of 4 or 8. • The student will verbalize steps in solving problems with a factor of 4 or 8 to explain mathematical thinking.	
Vocabulary	**Double It:** a strategy in which the student breaks apart the factor of 4 or 8 into groups of 2 and then uses the Distributive Property to multiply and add to find the product; for example, $4 \times 7 = 2 \times 7 + 2 \times 7$; $8 \times 7 = 2 \times 7 + 2 \times 7 + 2 \times 7 + 2 \times 7$	
Reviewed Vocabulary	dividend, division, divisor, equation, expression, factor, multiplication, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 192-201) • Hundreds chart (error correction)	• Student Booklet (pp. 97-101) • Whiteboard with marker (1 per student) • Counters (8 per student)

Preview

Say: **Today we will break apart the factors 4 and 8 to make multiplying by these factors quick.**

Engage Prior/Informal Knowledge — Time: 3 min

Review how to make groups of 2 and doubles facts. Give each student 8 counters and a whiteboard.

Say: **Count by 2s and write the multiples of 2 up to 10 on your board. Ready, count and write.** *(2, 4 … 10)*

The numbers 4 and 8 are both multiples of 2, which means we can equally share all the counters in groups of 2. Take your 8 counters and make groups of 2. How many groups of 2? *(4)* **What is the repeated addition expression?** *(2 + 2 + 2 + 2)*

This repeated addition problem has 2 doubles facts. A doubles fact is the same number added together, like 2 + 2 or 7 + 7. Name another example of a doubles fact. *(allow a variety of answers)*

Now make groups of 2 with 4 counters. How many groups? *(2)* **What is the repeated addition expression?** *(2 + 2)* **Is this a doubles fact?** *(yes)*

For the remaining time practice the following doubles facts on whiteboards: 6 + 6, 8 + 8, 14 + 14, 16 + 16, 32 + 32, 28 + 28.

Modeled Practice — Time: 8 min

1. The students will use an array to show the Double It strategy. The students will write the number family equations for each fact.

   Use *Modeled Practice Sheet #1*.

Say: **Look at the array. What is the multiplication expression for this array?** *(7 × 4)* **Write it on the box above the array.**

Think about groups of 2. How many groups of 2 can we make from 4 counters? *(2)* **Split the array so that it shows 2 arrays with 2 groups of 7 in each array.**

Check to see that students have split the array to show 7 × 2 and 7 × 2 on the dotted line. Write the products and sum as the lesson progresses. Have students complete their sheets following teacher directions.

Teacher Note
Following the Commutative Property, students may write the factors in either order. For example, 2 × 7 and 7 × 2 are both acceptable answers.

What is the multiplication expression for the first new array? *(7 × 2 or 2 × 7)* **Write it below the first array.**

What is the multiplication expression for the second new array? *(2 × 7 or 7 × 2)* **Write it below the second array.**

We split the array into 2 facts that we can solve quickly by skip counting or knowing our doubles facts. What is the repeated addition expression for 2 × 7? *(7 + 7)*

Step 1 of the *Double It* strategy is to double the other factor. What is the other factor? *(7)* **What is 7 + 7?** *(14)* **How many times do we need to double 7?** *(2)* **What is step 1 of the *Double It* strategy?** *(double the other factor)*

Write 14 below each multiplication expression.

What operation, addition or subtraction, do we use when combining parts together? *(addition)*

Step 2 is to double it again, meaning double the products or add the sums. What is step 2? *(double it again)*

Now add 14 + 14. What is 14 + 14? *(28)* **Write it.**

Then what is 7 × 4 or 4 × 7? *(28)*

What 3 numbers are in this number family? *(4, 7, and 28)* **Write it.**

What is 1 division equation for this number family? *(28 ÷ 4 = 7 or 28 ÷ 7 = 4)* **Write it.**

2. Have students use the Double It Strategy to solve facts with a factor of 8.

   Have students turn to ***Modeled Practice Sheet #2***.

Say: **Look at the array. What is the multiplication expression for this array?** *(7 × 8)* **Write it in the box above the array.**

How many groups of 2 are in 8? *(4)* **Make lines to separate the array into groups of 2 with 7 in each group. What is the multiplication expression for each new array?** *(7 × 2)* **Write it below each new array.**

Solving facts with 8 is similar to solving facts with a factor of 4. First we double the other factor. What is the other factor? *(7)* **How many times do we double 7 or solve 2 × 7?** *(4)* **What is 7 + 7 or 2 × 7?** *(14)* **Write 14 below each multiplication expression.**

Step 2 is to double it again. First add 14 + 14. What is 14 + 14? *(28)* **Write 28 on both lines.**

Step 3 is to double it the last time. What is step 3? *(double it the last time)*

What is the last doubles fact that we are solving? *(28 + 28)* **What is 28 + 28?** *(56)* **What is 8 × 7?** *(56)* **What 3 numbers are in this number family?** *(7, 8, 56)* **Write it. What is 1 division equation for this number family?** *(56 ÷ 7 = 8 or 56 ÷ 8 = 7)* **Write it.**

When solving facts with a factor of 8 we have to double it 3 times. When solving facts with a factor of 4 we double it 2 times. How many times do we double it for 8s? *(3)* **How many times do we double it for 4s?** *(2)*

We only need to use this strategy for a few 4 and 8 facts. Do we use the *Double It* strategy to solve when multiplying 4 times 1 and 8 times 1? *(no)* **What about 4 or 8 times 2, 5, or 10?** *(no)* **Why?** *(skip counting can be faster)* **What about 4 or 8 times 9, what strategy?** *(Make Ten Subtract the Factor)* **What about 4 or 8 times 6?** *(Break-apart 6)*

Watch For	**Students may struggle in solving doubles facts. Have students use a hundreds chart or write the answer to the doubles facts on a whiteboard to use during the lesson.**

Practice — Time: 8 min

Activity 1: Have students use the Double It strategy to solve problems. Have student identify the 3 numbers in the number family and then write the equations.

Have students turn to the *Practice Sheets* on pages 99 and 100. First, students will follow examples for the steps. Then students will complete the problems using the Double It Strategy.

While students are solving ask question such as:

- What is the other factor in the problem? *(answers will vary depending on the problem)*
- What is step 1? Step 2? Step 3? *(double the other factor; double it again; for 8s only, double the last time)*
- What 3 numbers are in this number family? *(answers will vary depending on the problem)*

Activity 2: Have students turn to the *Practice Sheets* on page 77. Students will fill in the fourth and eighth rows on the multiplication table with the 4s and 8s facts. Students will continue to refer to this multiplication table in the next lesson and complete the multiplication table as they learn more facts and strategies.

Ask questions such as:

- When we fill in the multiplication table today, how many facts do we use the Double It strategy to solve? *(7 total; 4 × 3, 4 × 4, 4 × 7, 4 × 8, 8 × 3, 8 × 7 and 8 × 8)*
- Name 2 multiples for 8. *(answers will vary)*
- Name 2 multiples for 4. *(answers will vary)*
- When will fill in the multiplication table, we are writing the product of 2 factors. What factor and 8 make a product of 64? *(8)*
- What factor and 3 make a product of 12? *(4)*
- What factor and 4 make a product of 32? *(8)*
- Can you use the multiplication table to find division facts? *(yes)*
- Explain how you can use the multiplication table to find division facts and give an example. *(on the multiplication table a product divided by a factor gives the quotient, such as 12 divided by 4 equals 3)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Relationships

Lesson 20

Solving for 7 Facts

Lesson Objectives	• The student will multiply factors by 7. • The student will use the Break-apart 7 strategy to solve facts with a factor of 7. • The student will verbalize steps in solving problems with a factor of 7 to explain mathematical thinking.	
Vocabulary	**Break-apart 7:** a strategy in which the student breaks apart the factor 7 into 2 and 5, and then uses the Distributive Property to multiply 2 and 5 by the other factor, and add the products; for example, $7 \times 7 = (5 \times 7) + (2 \times 7) = 49$	
Reviewed Vocabulary	dividend, division, divisor, equation, expression, factor, multiplication, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 202-213) • Whiteboard with marker	• Student Booklet (pp. 102-107) • Whiteboard with marker (1 per student)

Preview

Say: **Today we will learn a strategy to solve the 7 facts quickly.**

Engage Prior/Informal Knowledge — Time: 3 min

Review strategies for solving 9, 2, and 5 facts. Use the whiteboard and have students work along with you.

Say: **Write 9 × 7 on your whiteboard. What strategy can you use to solve this problem?** *(Make Ten Subtract the Factor)*

Solve the problem using the Make Ten Subtract the Factor strategy.

What is step 1? *(think of 9 as 10 – 1)* **Write 10.**

What is step 2? *(multiply 10 times the other factor)*

What is the other factor in the problem? *(7)* **Write x 7.**

What is 10 × 7? *(70)* **Write it.**

What is step 3? *(subtract the other factor)* **Write – 7.**

What is 70 – 7? *(63)*

What is 9 × 7? *(63)*

Make Ten Subtract the Factor is a helpful strategy to solve the 9 facts. What facts? *(9 facts)*

Now write 2 × 7 = on your whiteboard. What do you skip count by to solve? *(skip count by 2)*

What is the product of 2 × 7? *(14)*

Write 6 × 5 = on your whiteboard. What do you skip count by to solve? *(skip count by 5)*

What is the product of 6 × 5? *(30)*

These strategies we have reviewed help us solve multiplication facts.

Modeled Practice **Time: 8 min**

1. Students will solve multiplication facts using the 7s strategy.

   Have student turn to ***Modeled Practice Sheet #1***. The teacher and students will complete the steps together as the lesson progresses.

Say: **Look at the array. What is the multiplication expression for this array?** *(7 × 3)* **Write it in the box above the array.**

What are the 2 factors? *(7 and 3)*

What are some ways we can break apart 7? *(1 + 6, 2 + 5, 7 + 0, 4 + 3, 8 – 1, 10 – 3)*

Do we know a fast way to solve the 2 and 5 facts? *(yes)*

How? *(skip count by 2 and 5)* **Since we know how to solve the 2 and 5 facts by skip counting, we will break apart 7 into 5 + 2. Split the array so that it shows 2 groups and 5 groups of 7.**

What is the multiplication expression for the first new array? *(2 × 3)* **Write it below the first array.**

What is the multiplication expression for the second new array? *(5 × 3)* **Write it below the second array.**

This is step 1. What is step 1? *(break apart 7 into 5 and 2)* **Write 5 and 2 in the boxes.**

Step 2 is multiply 5 and 2 by the other factor. What is step 2? *(multiply 5 and 2 by the other factor)*

What is the other factor in this problem? *(3)* **Write 3 on both lines.**

What is 5 × 3? *(15)* **Write it below.**

What is 2 × 3? *(6)* **Write it below.**

Step 3 is to add the products. What is step 3? *(add the products)*

Now add 15 + 6. What is 15 + 6? *(21)*

What is 7 × 3? *(21)* **What is the corresponding multiplication equation?** *(3 × 7 = 21)*

What 3 numbers are in this number family? *(7, 3, and 21)*

What is the whole or the dividend in this number family? *(21)* **What are the division equations?** *(21 ÷ 7 = 3 or 21 ÷ 3 = 7)*

This is the *Break-apart 7* strategy. It can be used to solve 7 times 3, 4, 6, 7, and 8. Give me an example of a multiplication problem for which we could use the *Break-apart 7* strategy. *(7 × 3, 7 × 4, 7 × 6, 7 × 7, 7 × 8)*

Think about other 7s facts. Would we use this strategy to solve 1 × 7? *(no)* **Why?** *(faster to solve using the Identity Property)*

Think. What about 2 × 7? *(no)* **Why?** *(faster to skip count by 2)*

Think. What about 5 × 7? *(no, faster to skip count by 5)*

Think. What about 7 × 10? *(no, faster to skip count by 10)*

Think. What about 7 × 9? *(no, use the Make Ten Subtract the Factor strategy)*

2. Students will solve multiplication facts using the 7s strategy.

   Have the students turn to ***Modeled Practice Sheet #2***. Continue to work together as a group.

Say: **Look at the array. What is the multiplication expression for this array?** *(7 × 7)* **Write it in the box above the array.**

Can we use the *Break-apart 7* strategy to solve this problem? *(yes)*

Why? *(you are multiplying by 7)*

What are the 2 factors? *(7 and 7)*

What is step 1? *(break apart 7 into 5 and 2)*

Write 5 and 2 in the boxes.

What is step 2? *(multiply 5 and 2 by the other factor)* **Multiply 5 and 2 by the other factor.**

What is the other factor in this problem? *(7)*

Write 7 on both lines.

Split the array so that it shows 5 × 7 and 2 × 7.

What is 5 × 7? *(35)* **Write it below.**

What is 2 × 7? *(14)* **Write it below.**

Step 3 is to add the products. What is step 3? *(add the products, add 35 + 14)*

What is 35 + 14? *(49)*

What is 7 × 7? *(49)*

What 3 numbers are in this number family? *(7, 7, and 49)*

What is the whole or the dividend in this number family? *(49)*

What is the division equation? *(49 ÷ 7 = 7)*

Watch For

Students may forget to add the products together to solve the multiplication fact. For example, in the first problem of Modeled Practice, 7 × 3, the students may write 15 or 6 as the answer instead of 21 (15 + 6). Practice the steps of the strategy again and emphasize the last step of adding the products together. Show the problem using an array and have students push the pieces together to show that you have to add the products to find the whole amount of the array.

Practice **Time: 8 min**

Activity 1: Have students use the Break-apart 7 strategy to solve problems.

Have students turn to the ***Practice Sheets*** on pages 104 and 105. First, students will write the words in the blanks for the strategy steps. Then, have the students complete problems using the Break-apart 7 strategy.

While students are solving, ask question such as:

- What is the other factor in the problem? *(answers will vary depending on the problem)*
- What is Step 1? Step 2? Step 3? *(break apart 7 into 5 and 2, multiply 5 and 2 by the other factor, add the products)*

Activity 2: Have students turn to the ***Practice Sheets*** on page 77. Students will fill in the seventh row on the multiplication table with the 7s facts. After students complete the 7 facts, tell students to use the multiplication table to complete the missing rows of 3 facts, 11 facts, and 12 facts. For example, $2 \times 11 = 22$ can be used to fill in 11×2 in the twelfth row.

Ask questions such as:

- Name 2 multiples for 7. *(answers will vary)*
- What factor and 7 make a product of 49? *(7)*
- What factor and 7 make a product of 21? *(3)*
- What factor and 7 make a product of 42? *(6)*
- What are the 4 equations in the number family using 28, 7, and 4? *($4 \times 7 = 28$, $7 \times 4 = 28$, $28 \div 7 = 4$, $28 \div 4 = 7$)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.